

# Molecular evolution and phylogenetic tree

# Intra-species polymorphism

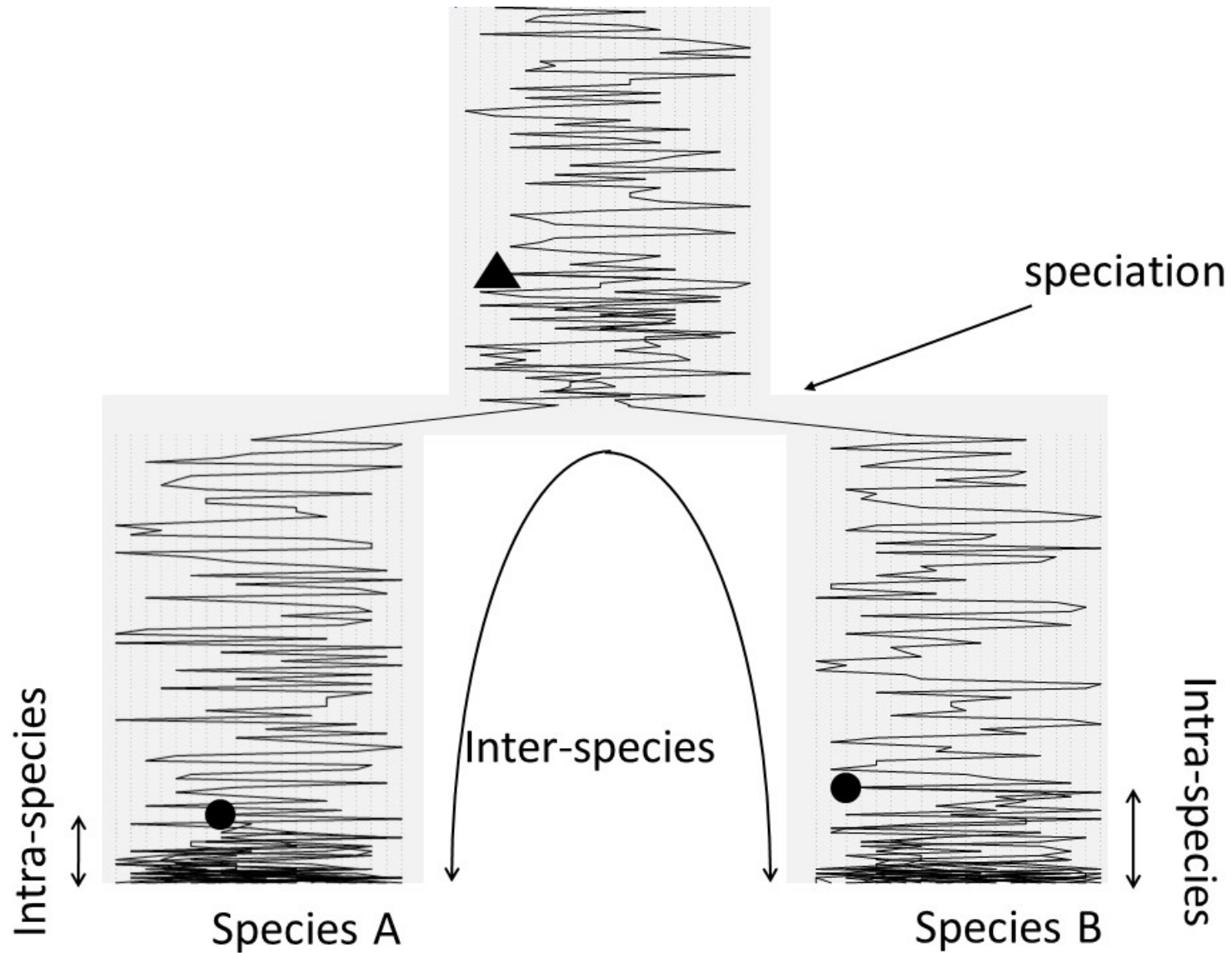
● monomorphic  
Δ weakly polymorphic  
○ polymorphic

association

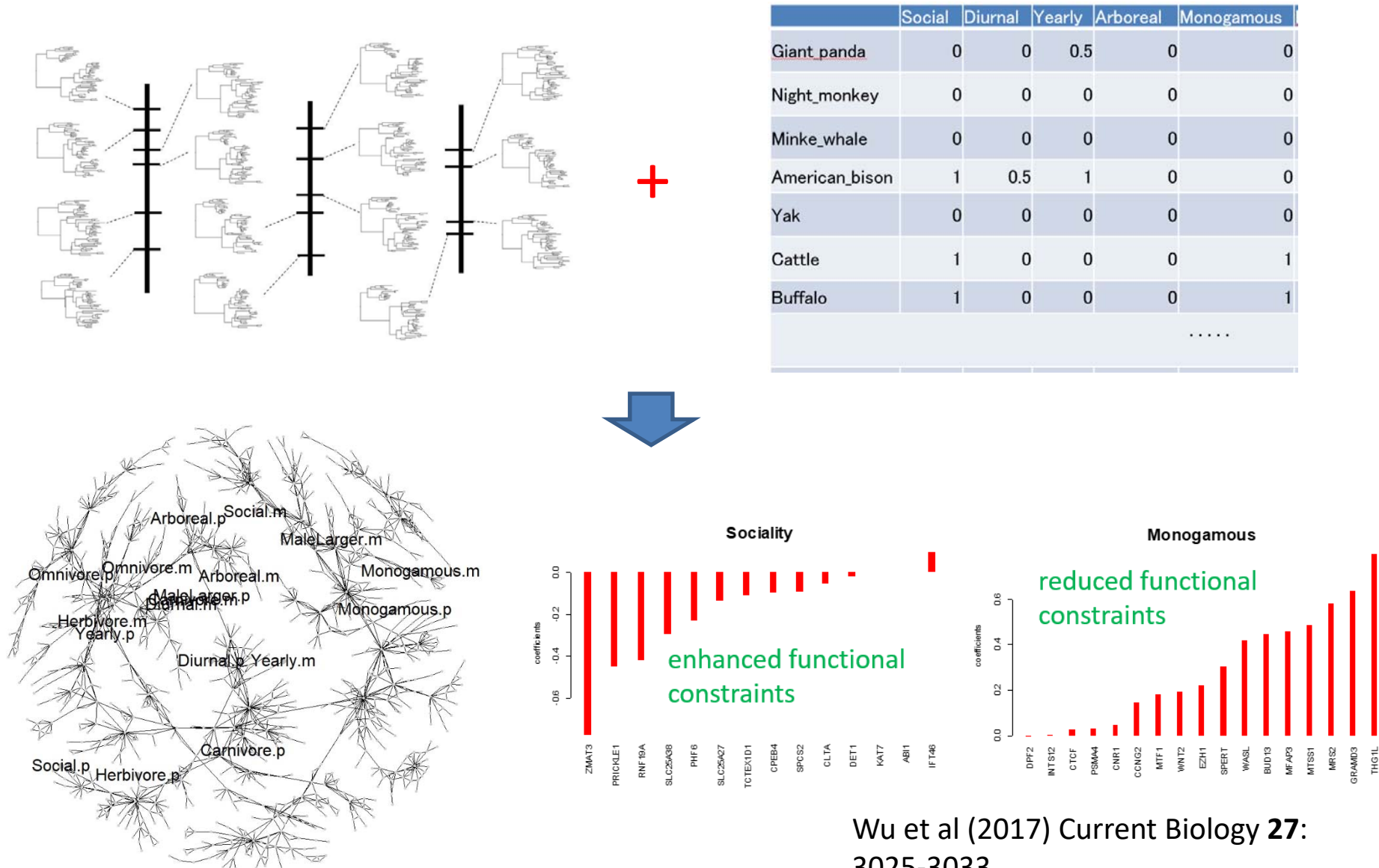
	●	Δ	●	●	●	○	●	●	●	●	●	○	●	●	phenotype	
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	T	C	T	G	A	G	G	A	G	...	S
...	T	T	A	G	A	C	C	T	G	A	G	G	A	G	...	S
...	T	T	A	G	A	T	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L
...	T	C	A	G	A	C	C	T	G	A	G	G	A	G	...	S
...	T	T	A	G	A	T	C	T	G	A	G	G	A	G	...	S
...	T	T	A	G	A	T	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	T	C	T	G	A	G	A	A	G	...	L
...	T	T	A	G	A	C	C	T	G	A	G	A	A	G	...	L

.....

# Inter-species diversity and intra-species polymorphism



# Bridging molecular evolution and phenotypic evolution



Wu et al (2017) Current Biology **27**: 3025-3033.

## Schedule of this course

Dates	Contents
15 May	Molecular evolution and phylogenetic tree
22 May	Rates of molecular evolution
29 May	Population structure and adaptation
5 May	Inferring traits evolution and selection

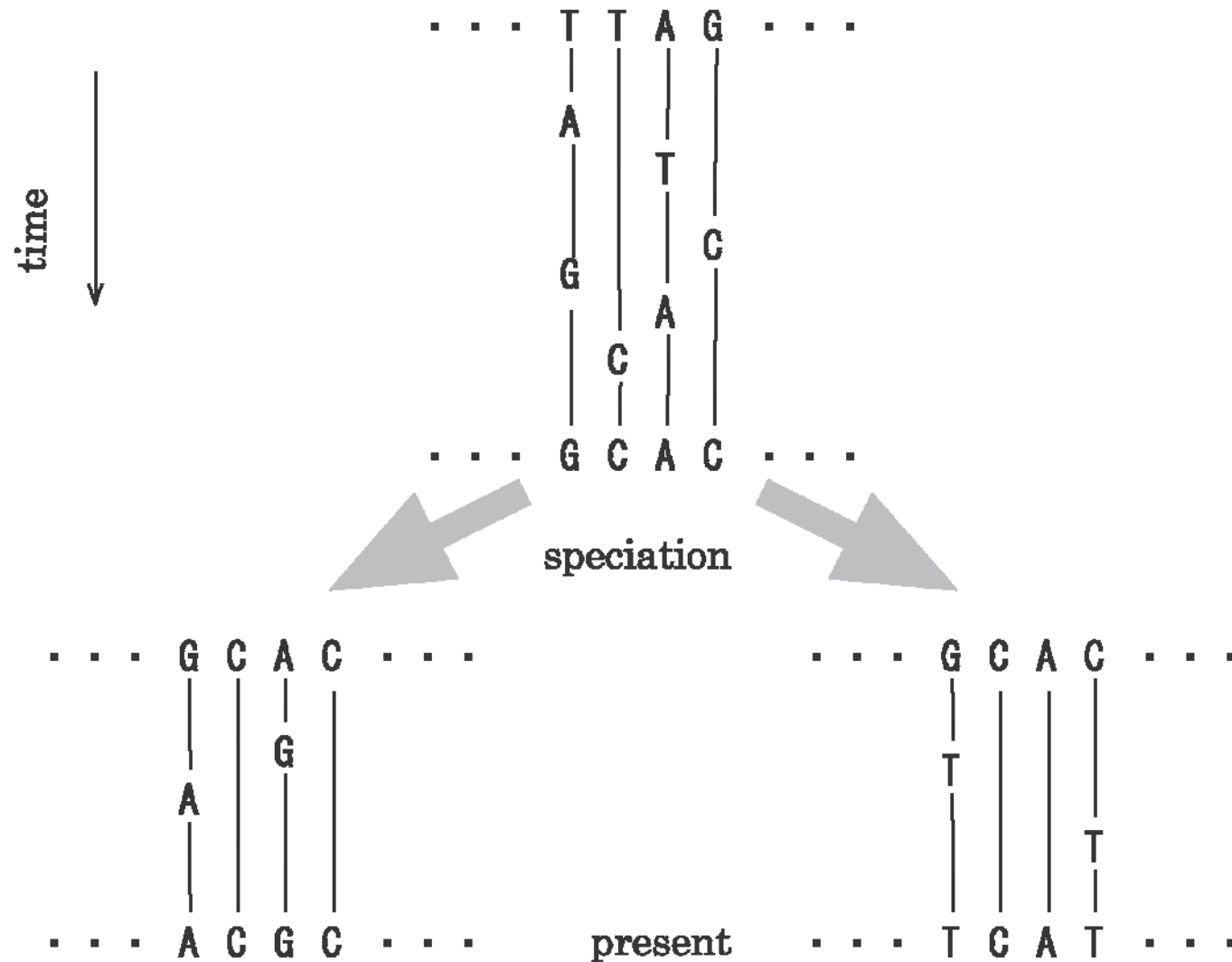
A dark blue circle is centered on a white background. Inside the circle, the words "Molecular" and "evolution" are written in white, stacked vertically.

Molecular  
evolution

## The cost of natural selection

Unless selection is very intense, the number of deaths needed to secure the substitution, by natural selection, of one gene for another at a locus, is independent of the intensity of selection. It is about 30 times the number of organisms in a generation. It is suggested that, in horotelic evolution, the mean time taken for each gene substitution is about 300 generations. This accords with the observed slowness of evolution.

Large collection of molecular evolution provides unbiased estimate of species trees

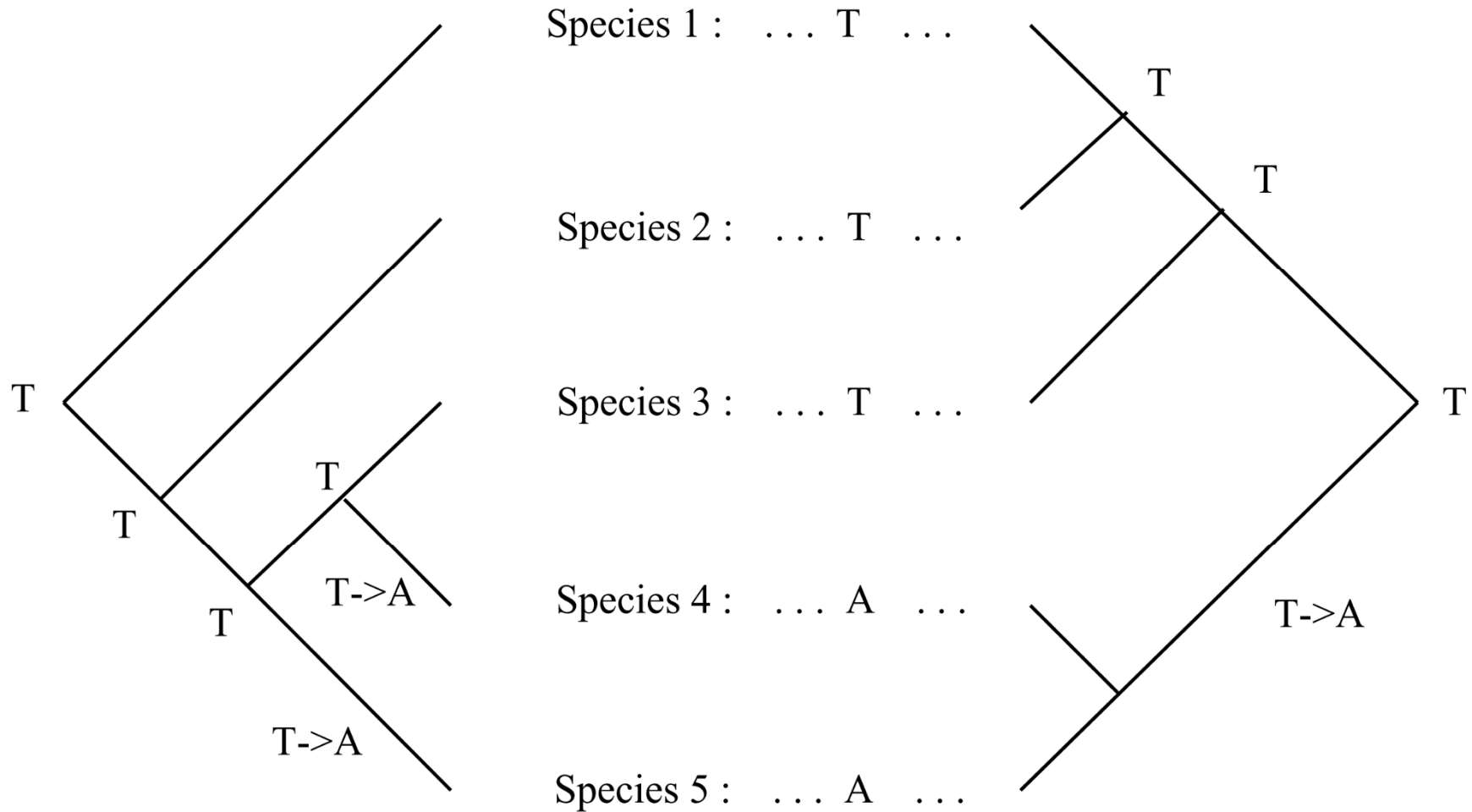




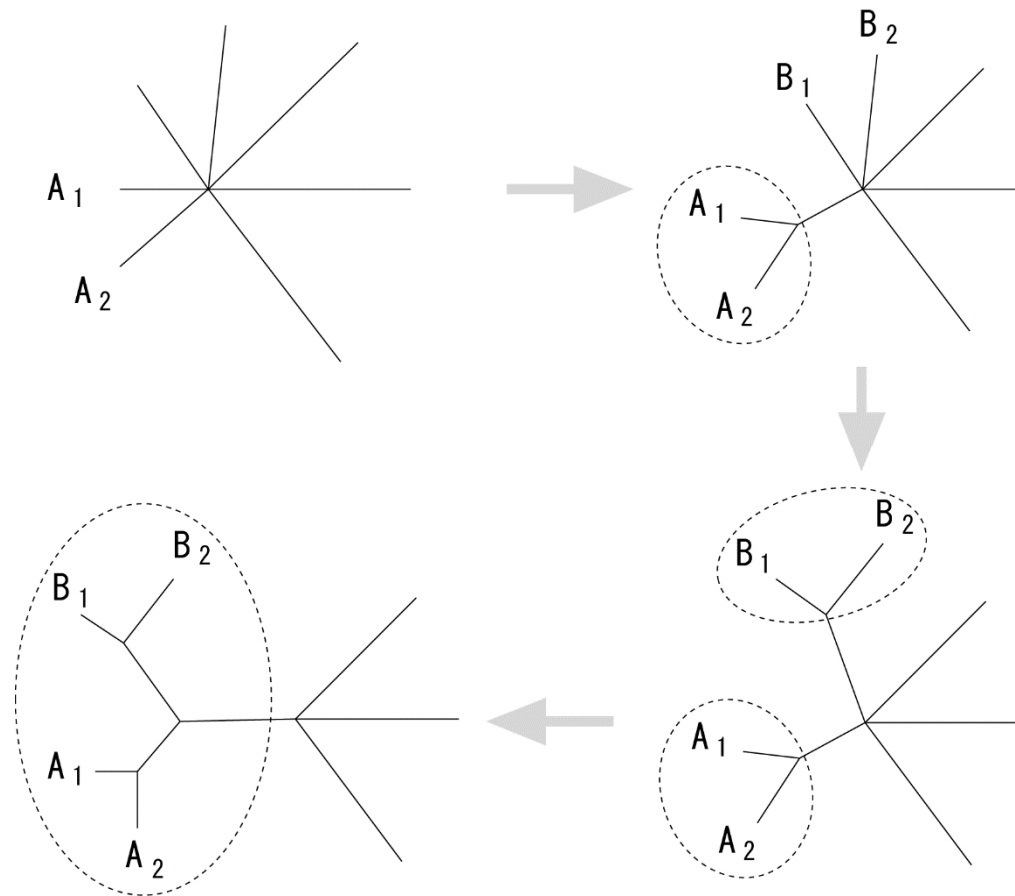
# Evolutionary Rate at the Molecular Level

Calculating the rate of evolution in terms of nucleotide substitutions seems to give a value so high that many of the mutations involved must be neutral ones.

# Reocnstructing the evolutionary history: Criteria of minimum evolution



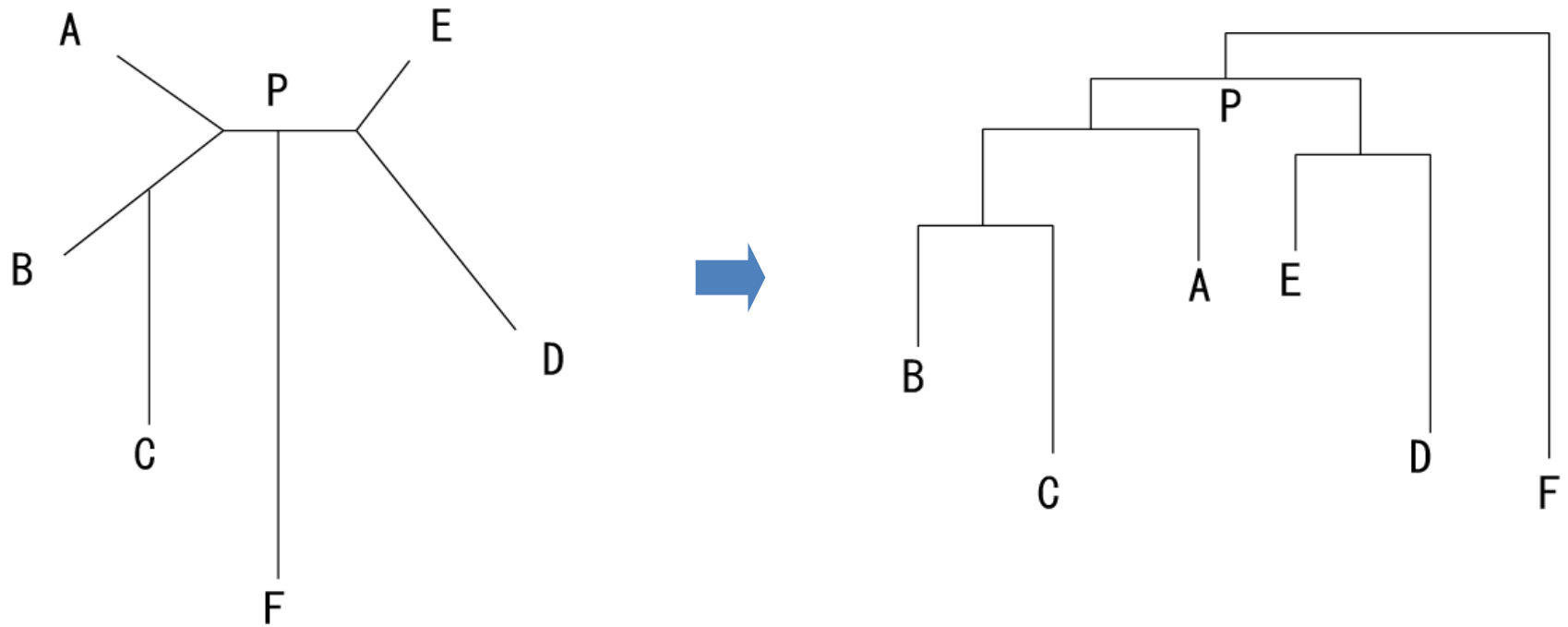
# Neighbor joining method



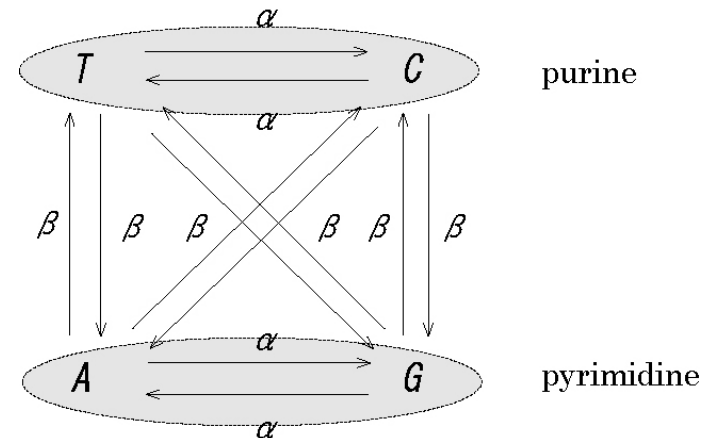
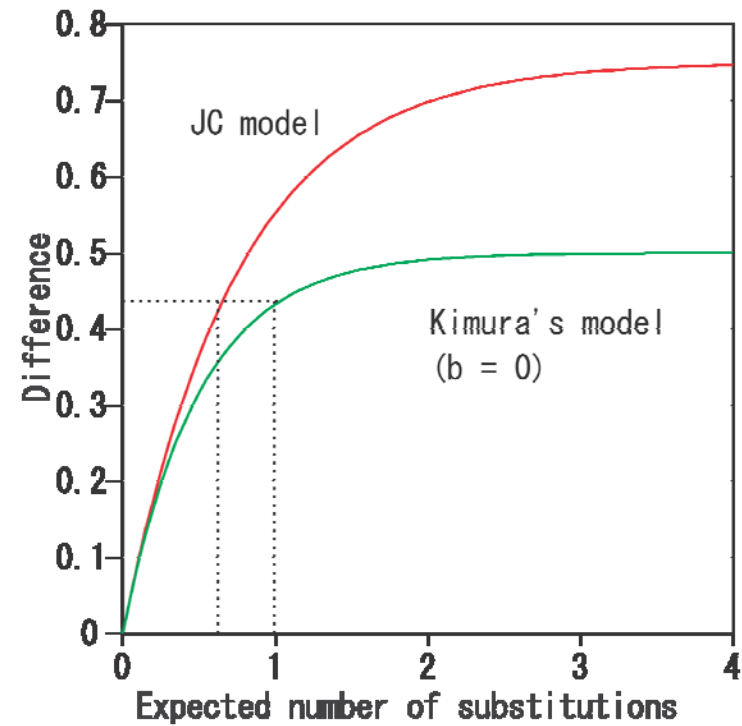
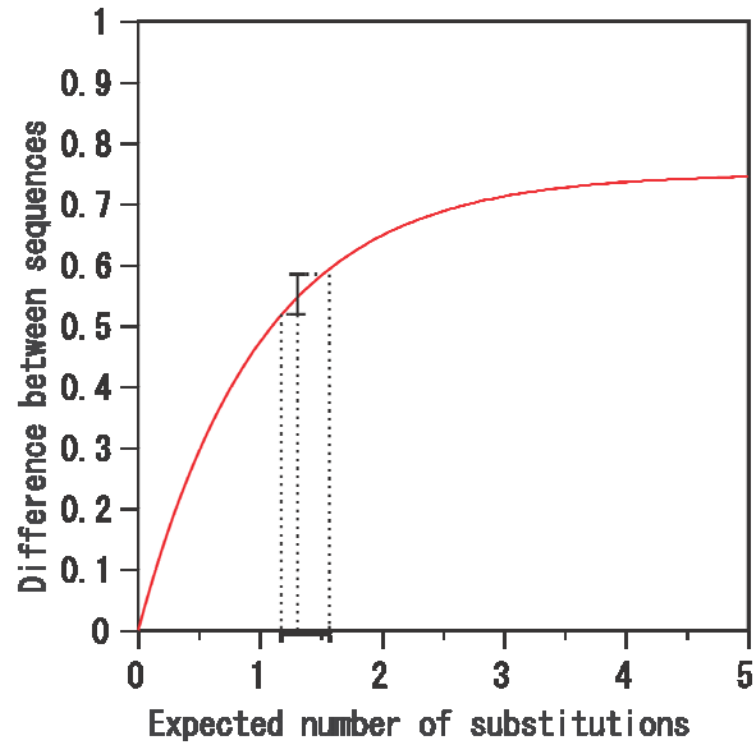
Minimizing the total branch length at each step

Saitou and Nei (1987) Mol Biol Evol. 4:  
406-425.

## Unrooted tree and rooting by an outgroup



## Distance based on statistical model of substitutions



## Estimating the pattern of substitutions from sequence comparison

..... **G T T C A C G G A T A C T G G T C T A C** .....  
..... **A T T C A T G A A T A C C C G C C T G C** .....



$p_{TT}, p_{TC}, p_{TA}, p_{TG}, p_{TT}, p_{CC}, p_{CA}, p_{CG}, p_{AA}, p_{AG}, p_{GG}$

Modeling the proportions of the pattern of sites

$$p_{ij} = p_i \times p_{i \rightarrow j}(t)$$

Felsenstein (1981)

$$p_{ii}(t) = \exp(-ut) + (1 - \exp(-ut)) \pi_i$$

$$p_{ij}(t) = (1 - \exp(-ut)) \pi_j \quad (i \neq j) ,$$

Kimura (1981)

$$\begin{aligned} p_{TT}(t) = p_{CC}(t) = p_{AA}(t) = p_{GG}(t) \\ = \frac{1}{4} + \frac{1}{4} \exp(-4\beta t) + \frac{1}{2} \exp(-2(\alpha + \beta)t) \end{aligned}$$

$$\begin{aligned} p_{TC}(t) = p_{CT}(t) = p_{AG}(t) = p_{GA}(t) \\ = \frac{1}{4} + \frac{1}{4} \exp(-4\beta t) - \frac{1}{2} \exp(-2(\alpha + \beta)t) \end{aligned}$$

$$\begin{aligned} p_{TA}(t) = p_{TG}(t) = p_{CA}(t) = p_{CG}(t) = p_{AT}(t) = p_{AC}(t) = p_{GT}(t) = p_{GC}(t) \\ = \frac{1}{4} - \frac{1}{4} \exp(-4\beta t) . \end{aligned}$$

# Models of nucleotide substitutions

Original	Mutant			
	A	T	C	G
1. Jukes-Cantor model:				
A	...	$\lambda$	$\lambda$	$\lambda$
T	$\lambda$	...	$\lambda$	$\lambda$
C	$\lambda$	$\lambda$	...	$\lambda$
G	$\lambda$	$\lambda$	$\lambda$	...
2. Felsenstein model:				
A	...	$\pi_T \lambda$	$\pi_C \lambda$	$\pi_G \lambda$
T	$\pi_A \lambda$	...	$\pi_C \lambda$	$\pi_G \lambda$
C	$\pi_A \lambda$	$\pi_T \lambda$	...	$\pi_G \lambda$
G	$\pi_A \lambda$	$\pi_T \lambda$	$\pi_C \lambda$	...
3. Kimura model:				
A	...	$\beta$	$\beta$	$\alpha$
T	$\beta$	...	$\alpha$	$\beta$
C	$\beta$	$\alpha$	...	$\beta$
G	$\alpha$	$\beta$	$\beta$	...
4. Hasegawa <i>et al.</i> model:				
A	...	$\pi_T \beta$	$\pi_C \beta$	$\pi_G \alpha$
T	$\pi_A \beta$	...	$\pi_C \alpha$	$\pi_G \beta$
C	$\pi_A \beta$	$\pi_T \alpha$	...	$\pi_G \beta$
G	$\pi_A \alpha$	$\pi_T \beta$	$\pi_C \beta$	...
5. Tamura-Nei model:				
A	...	$\pi_T \beta$	$\pi_C \beta$	$\pi_G \alpha_1$
T	$\pi_A \beta$	...	$\pi_C \alpha_2$	$\pi_G \beta$
C	$\pi_A \beta$	$\pi_T \alpha_2$	...	$\pi_G \beta$
G	$\pi_A \alpha_1$	$\pi_T \beta$	$\pi_C \beta$	...
6. Rzhetsky-Nei model:				
A	...	$\beta_2$	$\beta_3$	$\alpha_4$
T	$\beta_1$	...	$\alpha_3$	$\beta_4$
C	$\beta_1$	$\alpha_2$	...	$\beta_4$
G	$\alpha_1$	$\beta_2$	$\beta_3$	...



# Modeling the pattern of molecular evolution and the transition probabilities

Felsenstein (1981)

$$p_{ii}(t) = \exp(-ut) + (1 - \exp(-ut)) \pi_i$$

$$p_{ij}(t) = (1 - \exp(-ut)) \pi_j \quad (i \neq j) ,$$

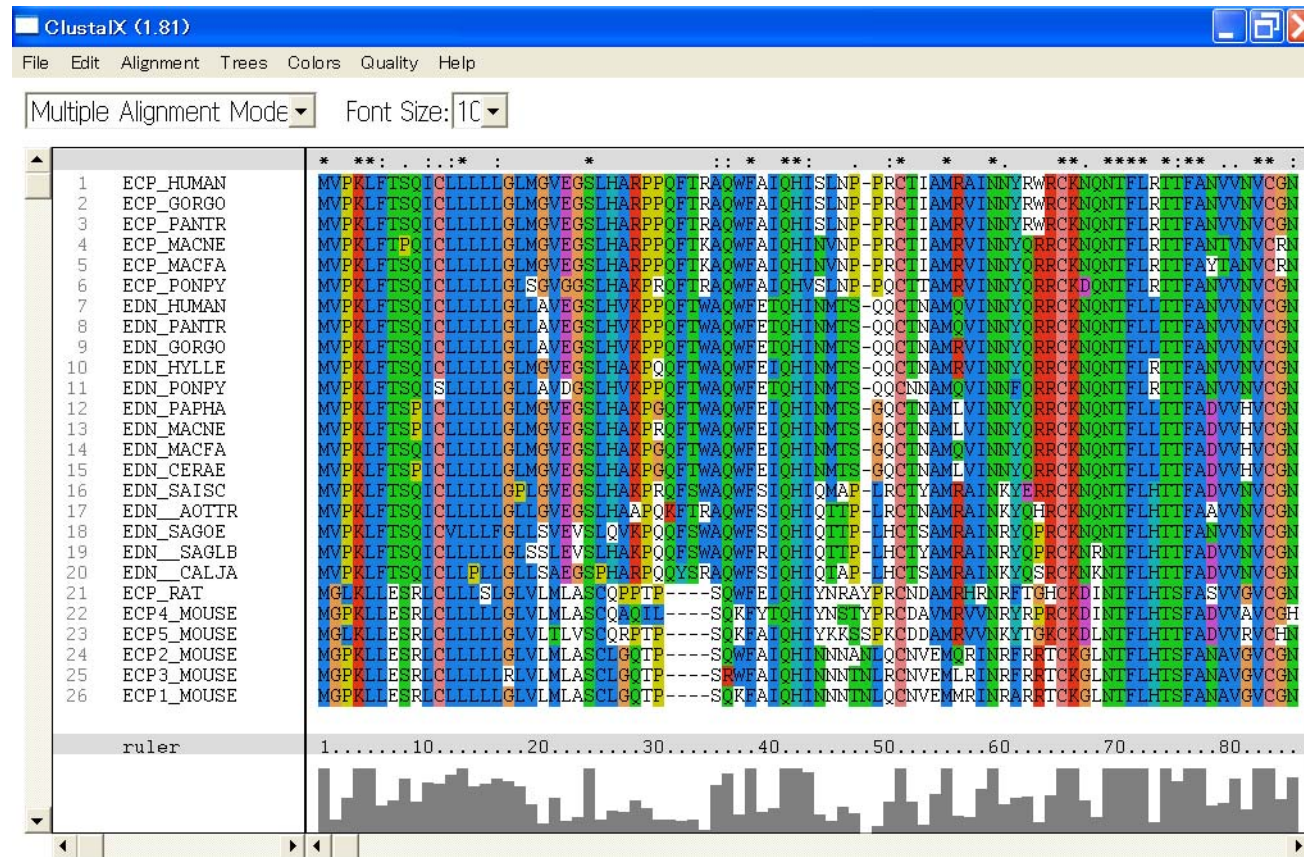
Kimura (1980)

$$\begin{aligned} p_{TT}(t) = p_{CC}(t) = p_{AA}(t) = p_{GG}(t) \\ = \frac{1}{4} + \frac{1}{4} \exp(-4\beta t) + \frac{1}{2} \exp(-2(\alpha + \beta)t) \end{aligned}$$

$$\begin{aligned} p_{TC}(t) = p_{CT}(t) = p_{AG}(t) = p_{GA}(t) \\ = \frac{1}{4} + \frac{1}{4} \exp(-4\beta t) - \frac{1}{2} \exp(-2(\alpha + \beta)t) \end{aligned}$$

$$\begin{aligned} p_{TA}(t) = p_{TG}(t) = p_{CA}(t) = p_{CG}(t) = p_{AT}(t) = p_{AC}(t) = p_{GT}(t) = p_{GC}(t) \\ = \frac{1}{4} - \frac{1}{4} \exp(-4\beta t) . \end{aligned}$$

# Multiple alignment identifies rate heterogeneity among sites



# A note on maximum likelihood and parsimony: the case of complete observation

$n_1, n_2, \dots, n_k$ : occurrence (1) / non-occurrence (0) of evolutionary events along the branches 1,2,...,k

- Assuming no multiple evolutionary events along a branch

- Likelihood

$$L(b|n_1, n_2, \dots, n_k) = \prod_{i=1}^k b^{n_i} (1 - b)^{1-n_i}$$

$b_1 = b_2 = \dots = b_k = b$ : probability of occurrence along a branch

- Assuming rare occurrence:  $\lambda = kb$

$$L(\lambda|n_T) = e^{-\lambda} \frac{\lambda^{n_T}}{n_T!}$$

$$n_T = n_1 + n_2 + \dots + n_k:$$

the number of evolutionary events assuming tree  $T$

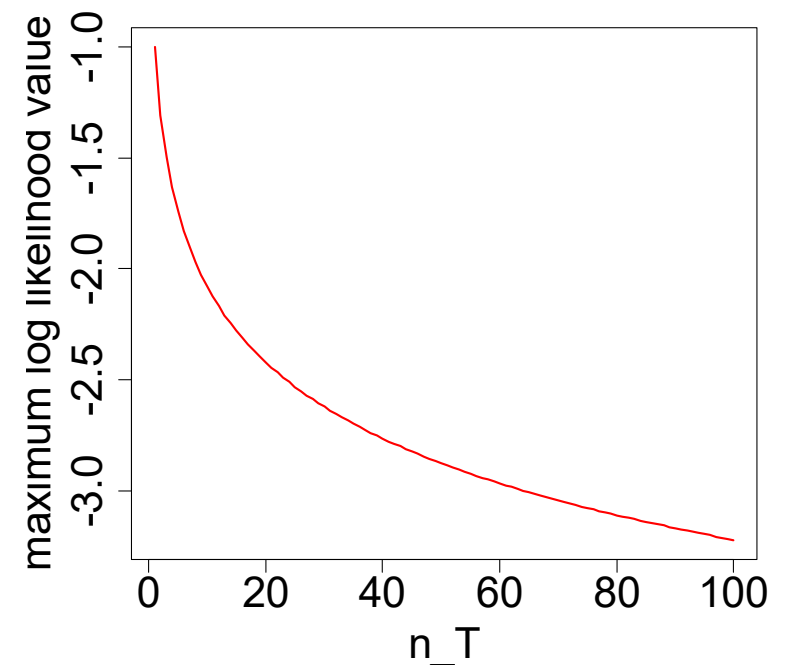
# A note on maximum likelihood and parsimony: the case of complete observation

- Maximum likelihood estimate and parsimony

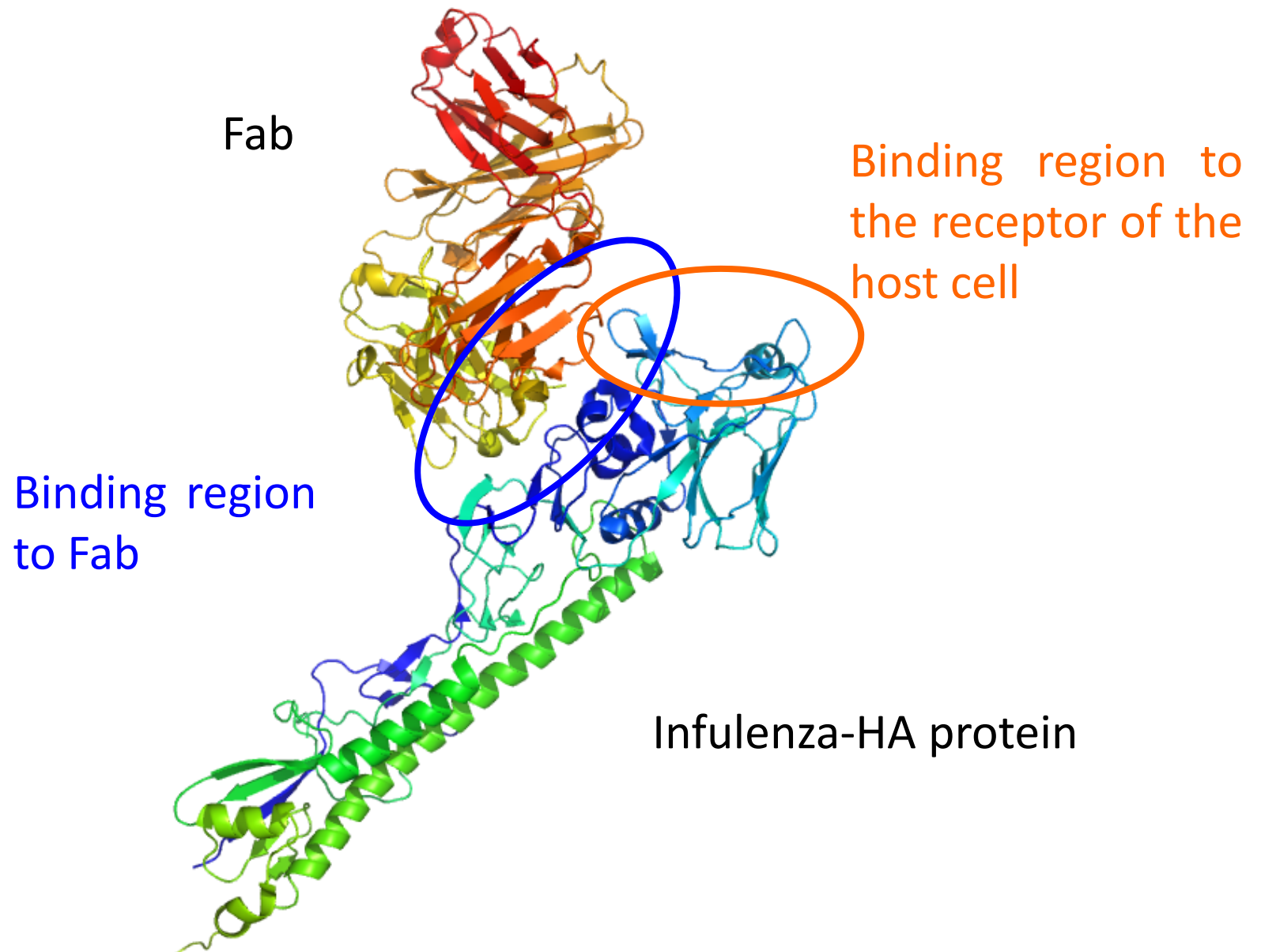
$$\hat{\lambda} = n_T$$

$$\begin{aligned}\log L(\hat{\lambda}|n_T) &= -\hat{\lambda} + n_T \log \hat{\lambda} - \sum_{i=1}^{n_T} \log i \\ &= -n_T + n_T \log n_T - \sum_{i=1}^{n_T} \log i\end{aligned}$$

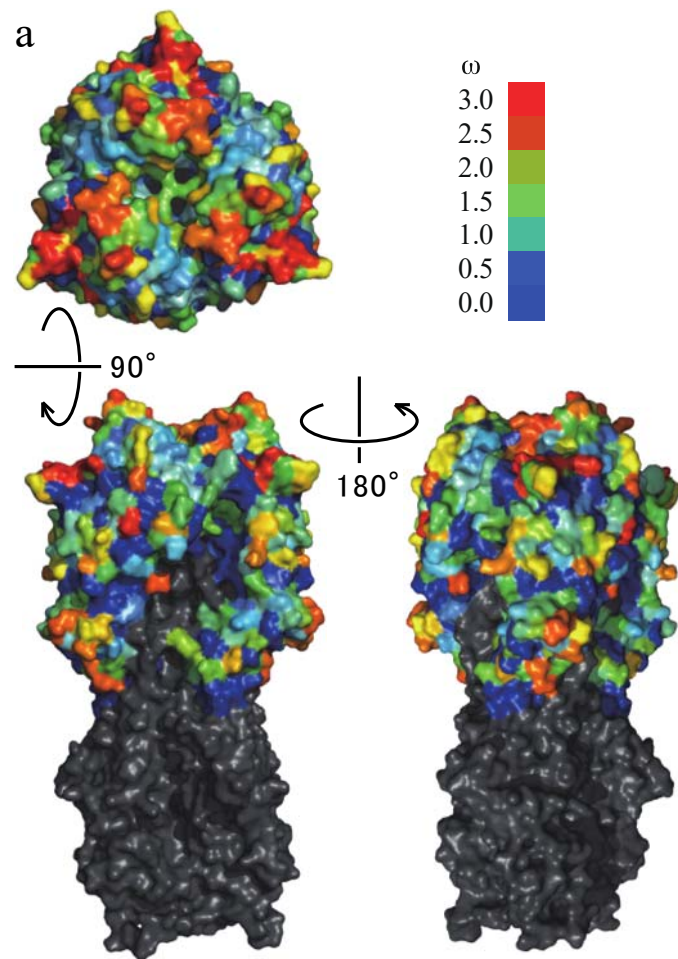
Likelihood approach is consistent with the criterion of minimum evolution



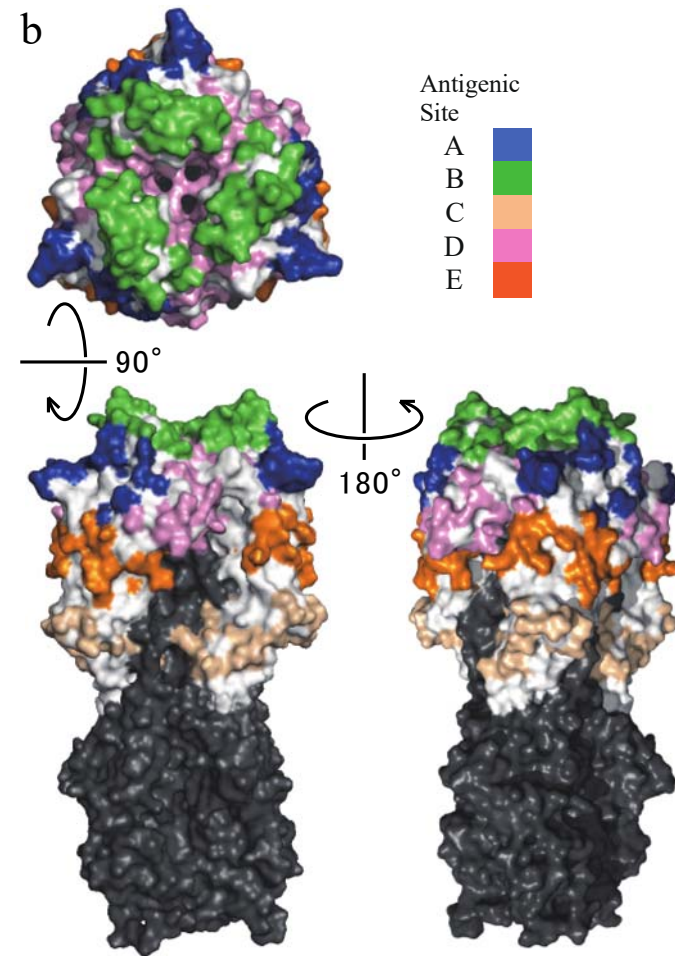
Selection pressure varies among the region of the protein



# Regions under the diversifying selection and antigenic sites



The spatial distribution of selection pressure

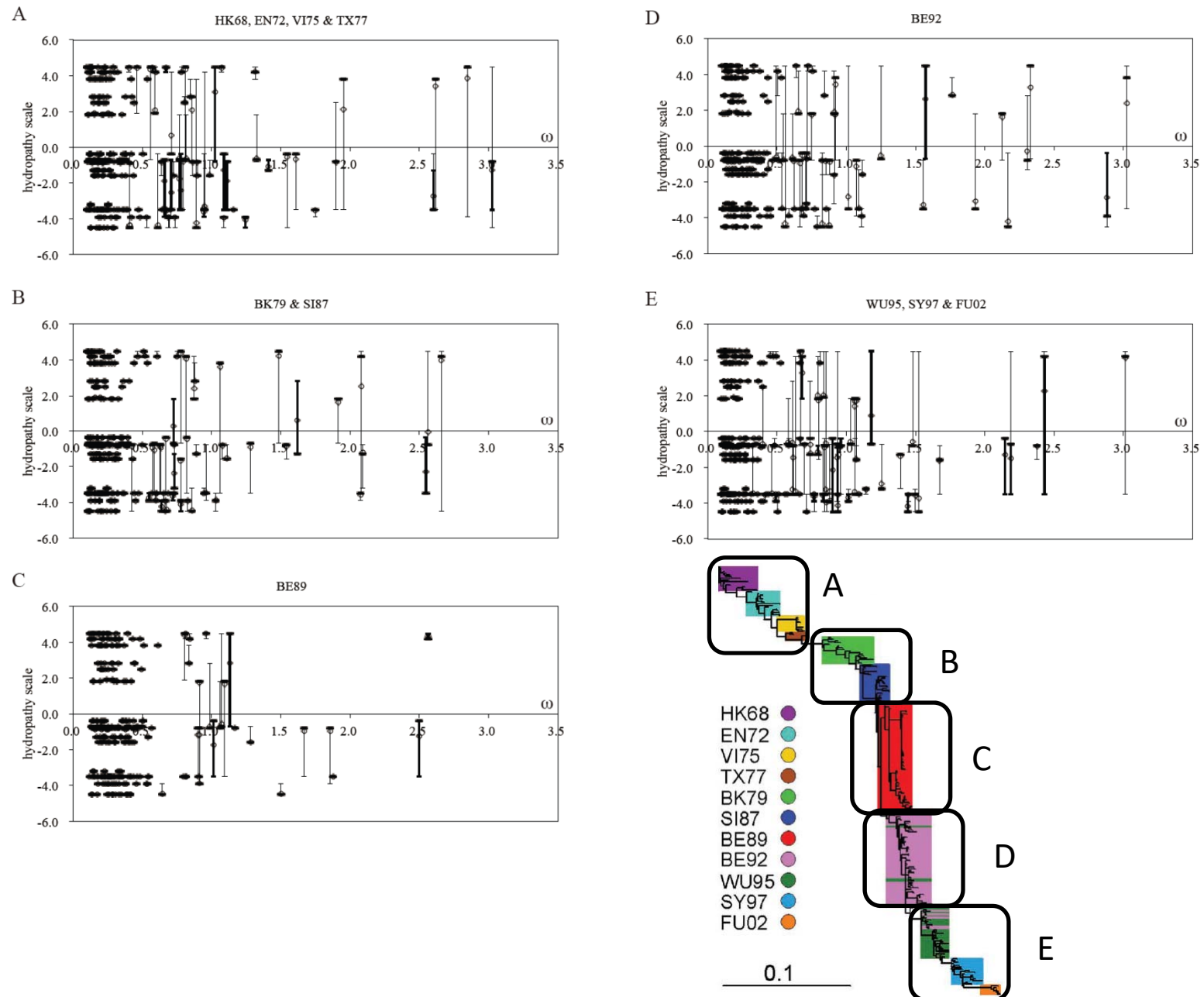


The distributions of the antigenic sites A to E.

Watabe and Kishino (2013) Mol Biol Evol. 30: 2714-2722.



# Changes in chemico-physical feature and the $\omega$ values of substitutions



# Rate of molecular evolution, mutation rate, and fixation rate

- Fitness of mutations: advantageous (rare), neutral, deleterious
- Mutations deleted from the populations are not observed by comparing the genomes
- Molecular evolution: mutations fixed to the populations

$$r = \mu \times 2N \times f$$

$$= \mu \times 2N \times p \times \frac{1}{2N}$$

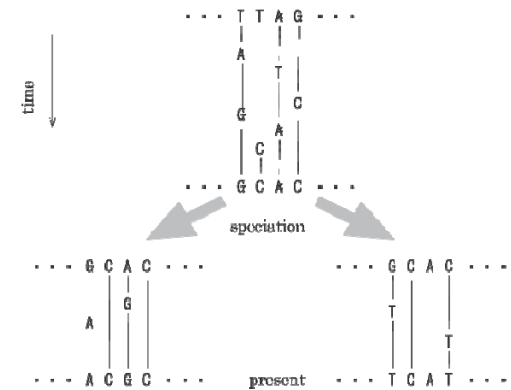
$$= \mu p$$

neutral

$p$

deleterious

$1 - p$



- Rate of molecular evolution is the product of **mutation rate** and **proportion of neutral mutations**

Generation length, mutagens

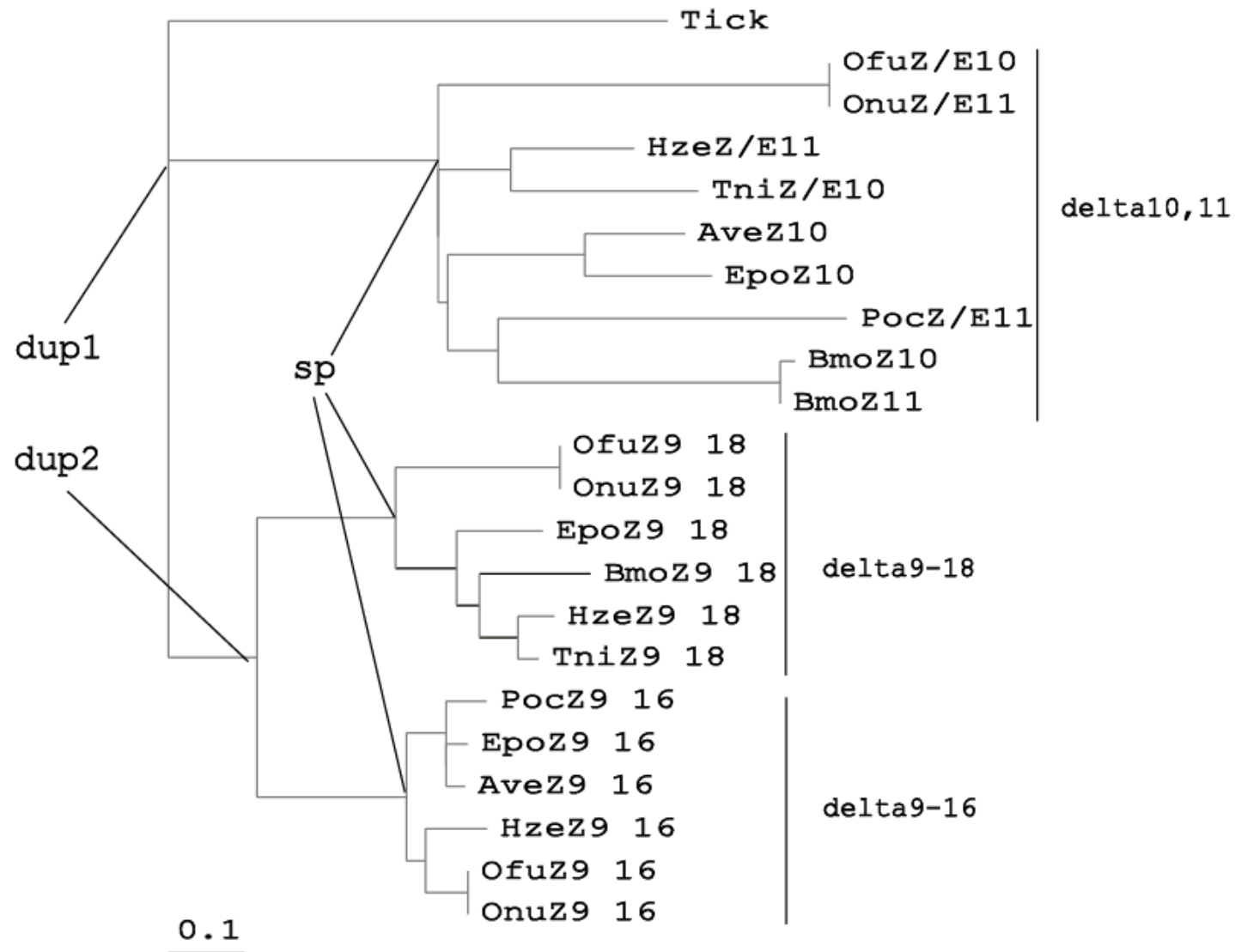
(affects on the whole genome)

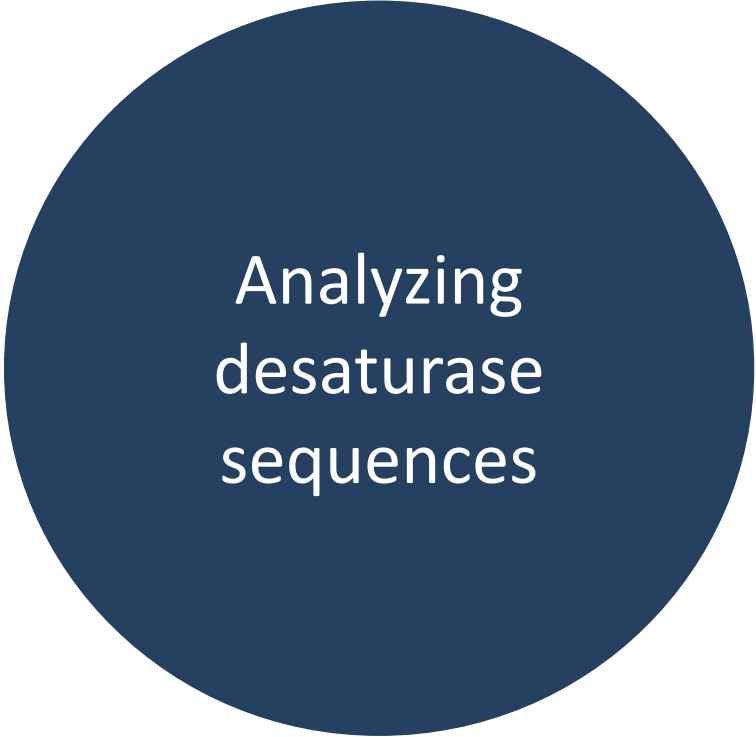
Functional constraints, diversifying selection

(vary among genes in the genome)



# Desaturase and the origin of sex pheromone



A dark blue circle is centered on a white background. Inside the circle, the text "Analyzing desaturase sequences" is written in white, sans-serif font, arranged in three lines.

Analyzing  
desaturase  
sequences

# Amino acid sequences of desaturase (fasta format)

```
>HzeZ9_18
MPPQGQTGGSWVLYETDAVNEDTDAPVIVPPSAEKREWKIVWRNVILMGMLHIGGVYGAY
LFLTAMWRTCIFAVVLYICSLGITAGAHRLWAHKSYPKARLPLRLMLTLFNTLAFQDAV
IDWARDHRMHHKYSETDADPHNATRGFFFAHVGWLLVRKHPQIKAKGHTIDLSDLKSDPI
LRFQKKYYLFLMPLVCFILPCYIPT-LWGESLWNAYFVCSIFRYVYVLNVTWLVNSAAHL
WGAKPYDKNINPVETRPVSLVVLGEGFHNYHHTFPWDYKTAELGDYSLNLTCLFIDTMAA
IGWAYDLKTVSTDVIQKRVKRTGDGSHPVWGWDDHEVHQADKKLAAIINPEKT

>TniZ9_18
MPPQGQTGGSWVLYETDAVNEDTDAPVIVPPSAEKREWKIVWRNVILMGMLHIGGVYGAY
LFLTAMWLTDLFAFFLYLCSGLGITAGAHRLWAHKSYPKARLPLRLMLTLFNTLAFQDAV
IDWARDHRMHHKYSETDADPHNATRGFFFSHVWLLVRKHPQIKAKGHTIDLSDLKSDPI
LRFQKKYYLTLMPLICFILPSYIPT-LWGESAFNAFFVCSIFRYVYVLNVTWLVNSAAHL
WGSKPYDKNINPVETRPVSLVVLGEGFHNYHHTFPWDYKTAELGDYSLNFTKMFIDFMAA
IGWAYDLKTVSTDVIQKRVKRTGDGSHAVWGWDDHEVHQEDKKLAAIINPEKT

>BmoZ9_18
MPPQQRKQEASWVLYEADANNLPEDAPPHVPPSAEKRPWKIVWRNVILFFILHVGGVYGGY
LFLFKAMWRTSIFAIFLYLCSGLGITAGAHRLWAHKSYPKARLPLRILLTIFNTIAFQDAV
VDWARDHRMHHKYSETDADPHNATRGFFFSHIGWLLLRKHPEIKAKGHTVDVNELRNDPI
LRFQKKYYQILMPLACFIMPTYVPT-LWGETVWNSFYVCAIFRYVYVLNITWLVNSAAHM
WGSKPYDKNINPVETRPVSLVVLGEGFHNYHHTFPWDYKTAELGDYSLNLSKLFIDFMAK
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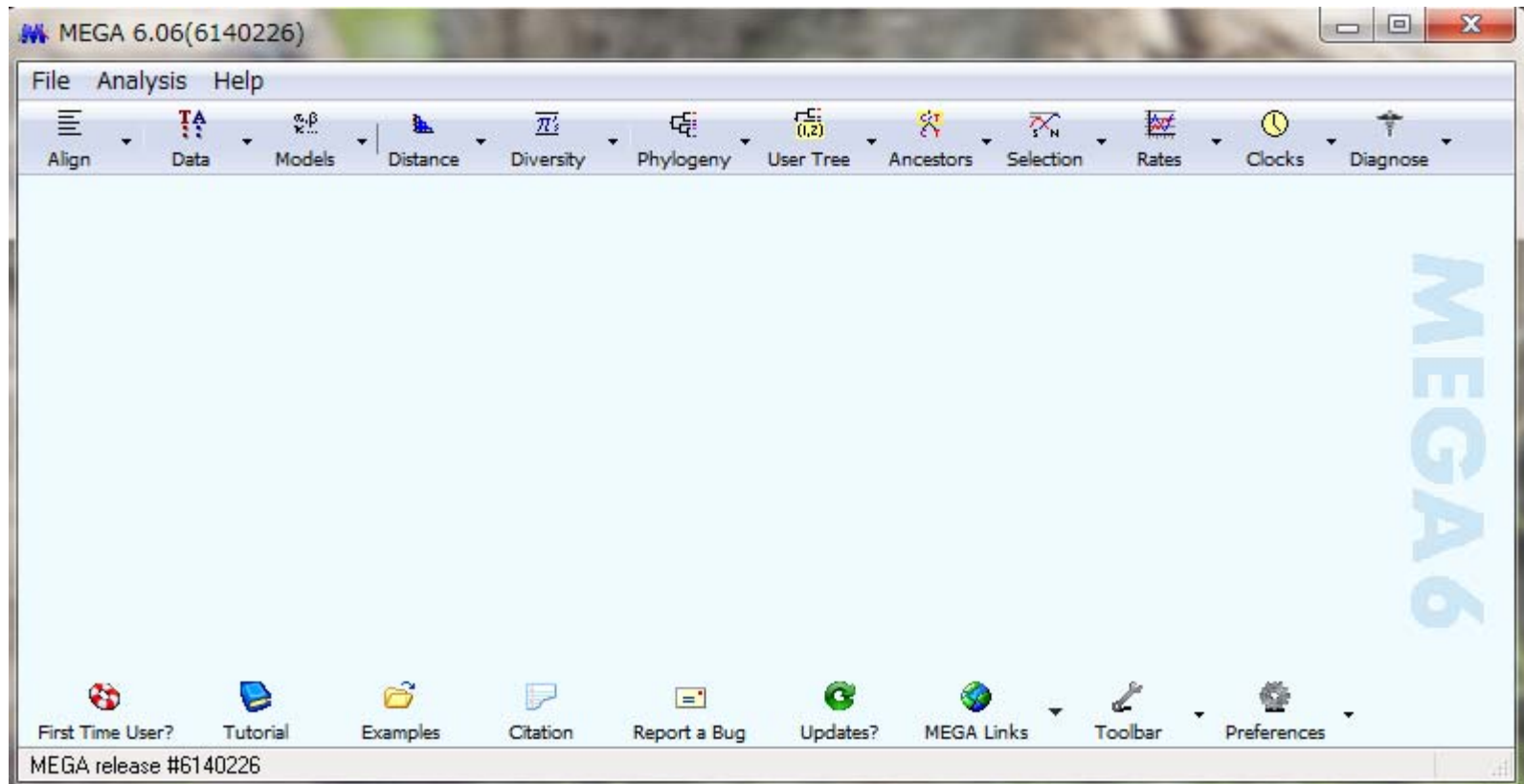
>EpoZ9_18
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LDWARDHRMHHKYSETDADPHNATRGFFFSHVWLLVRKHPQIKAKGHTIDMSDLCSDPV
LRFQKKYYLTLMPLFCFILPTYIPT-LWGESLWNAYFVAIFRYCYVLNVTWLVNSAAHK
WGDRPYDKNINPVETKPVSLVVFEGEGFHNYHHTFPWDYKTAELGGYSLNISKLFIDTMAK
IGWAYDLKSVSPDIVEKRVKRTGDGSHEVWGWDDKDVPAEQKAAATIIINPEKT

. . . . .
```

desaturase.fasta

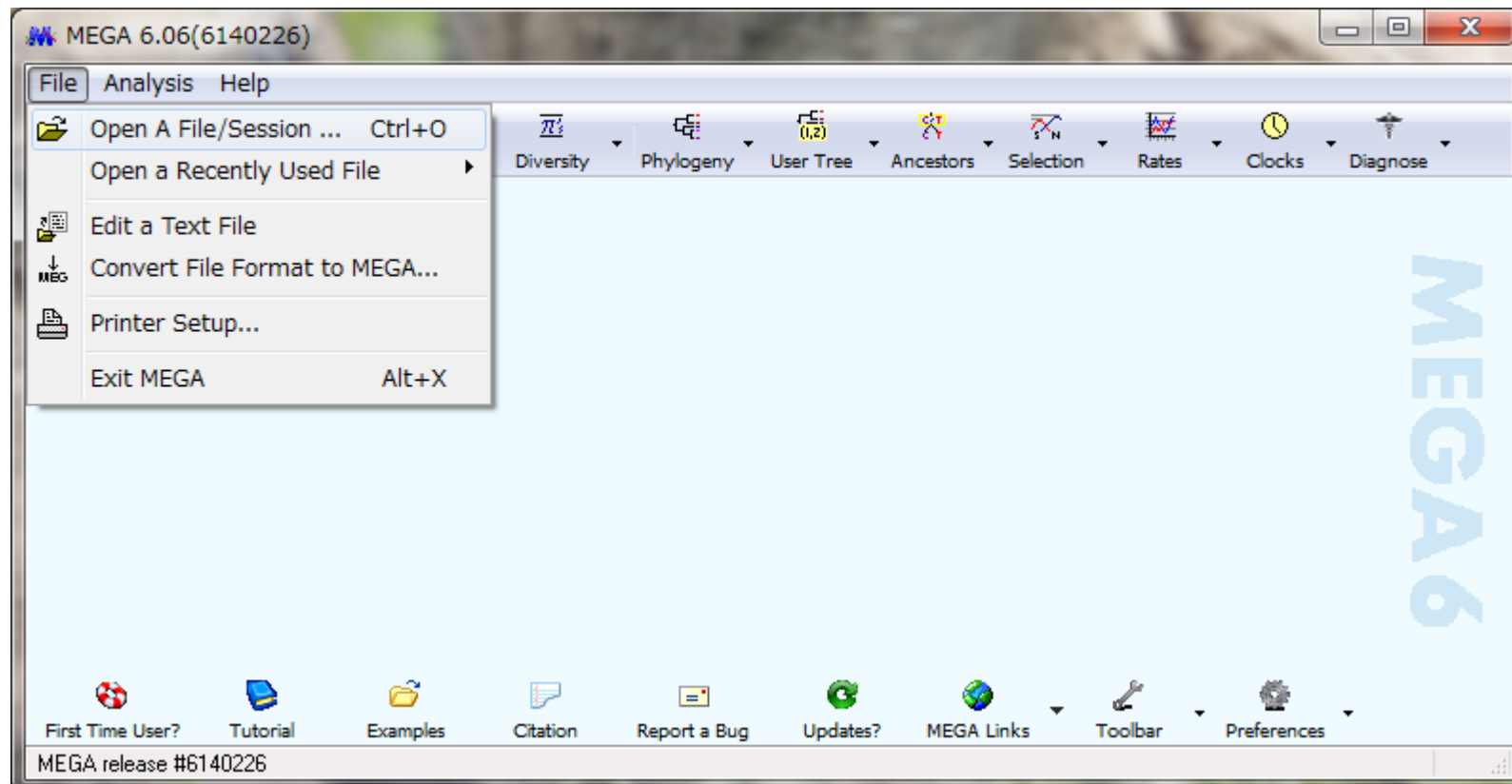
# Start MEGA

Double click the shortcut



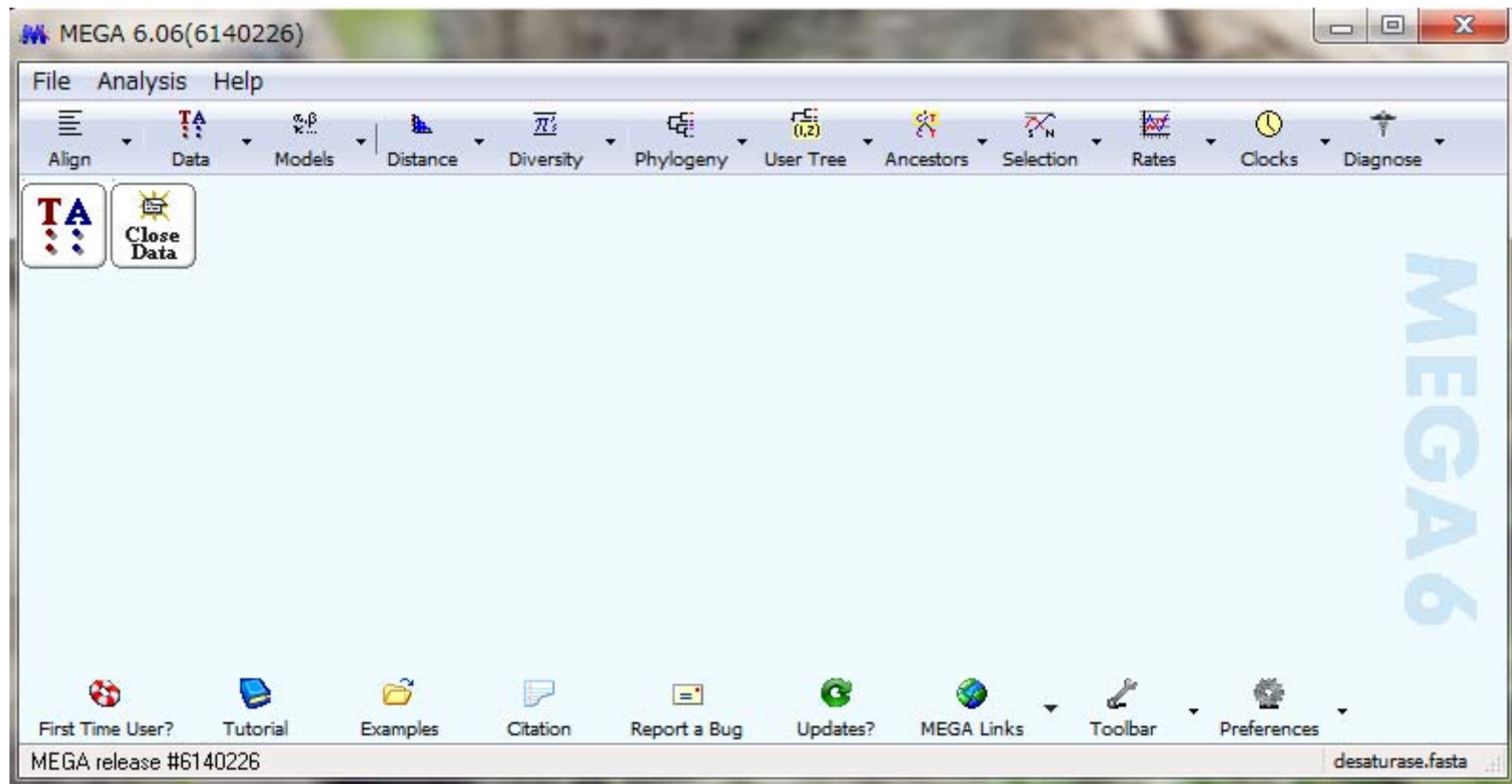
# Open the file of amino acid sequences

[File][Open A File/Session]



# Open the file of amino acid sequences

"Analyze" -> "Protein Sequence"



# View the data

Double click



MEGA 6.06(6140226)

File Analysis

Align Data

MEGA logo icon

Close Data

First Time User?

MEGA release #614

M6: Sequence Data Explorer

Data Display Search Groups Highlight Statistics Help

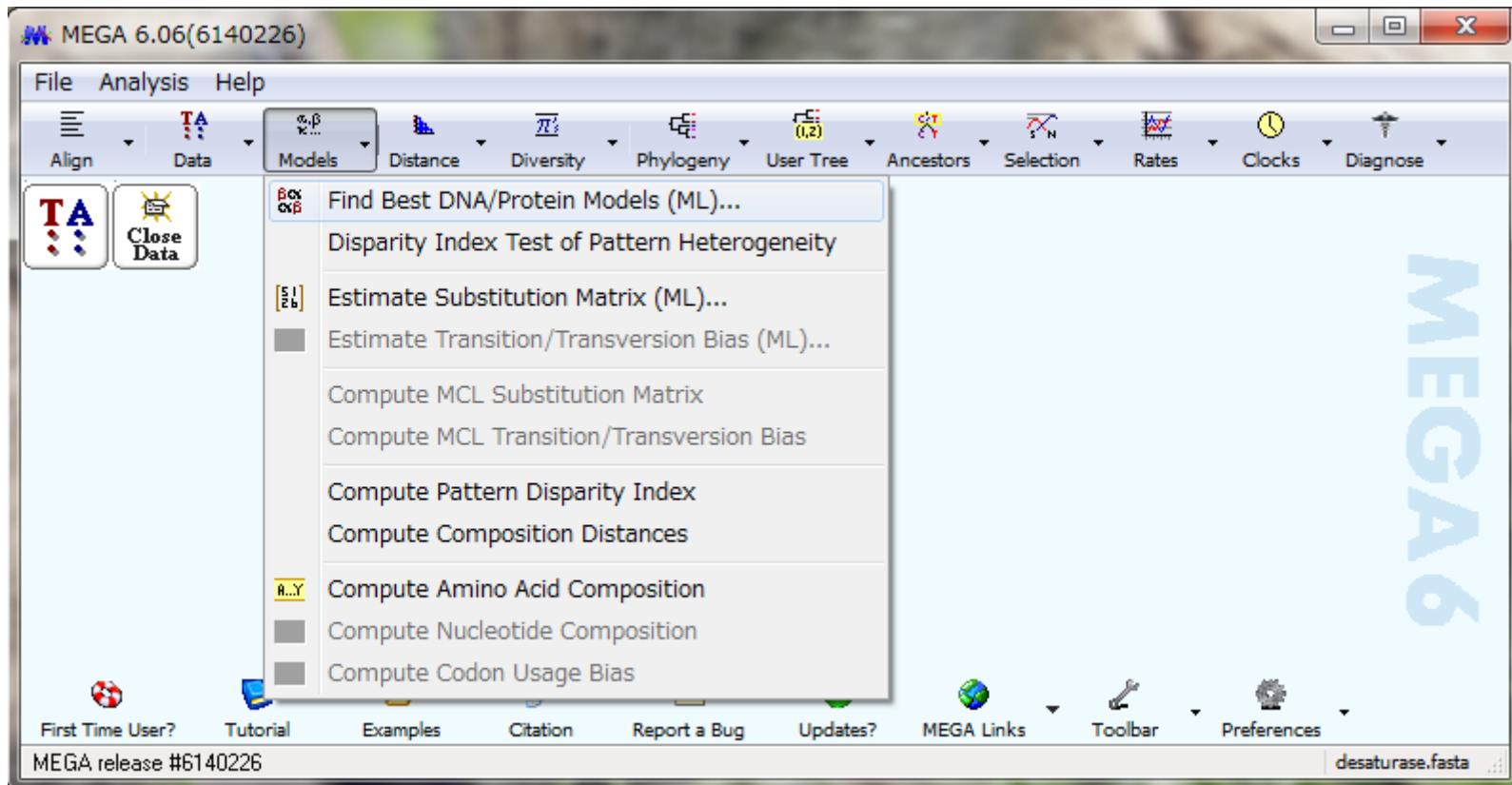
UUC C V Pi S 0 2 4 Special

Name	Group	M	P	P	Q	G	Q	T	G	S	W	V	L	Y	E	T	D	A	V	N	E	D	T	D	A	P	V	I	V	P	P	S	A	E
1. HzeZ9_18																																		
2. TniZ9_18																																		
3. BmoZ9_18																																		
4. EpoZ9_18																																		
5. OfuZ9_18																																		
6. OnuZ9_18																																		
7. EpoZ9_16																																		
8. PocZ9_16																																		
9. AveZ9_16																																		
10. OfuZ9_16																																		
11. OnuZ9_16																																		
12. HzeZ9_16																																		
13. BmoZ10																																		
14. BmoZ11																																		
15. AveZ10																																		
16. EpoZ10																																		
17. OfuZ/E10																																		
18. OnuZ/E10																																		

1/353 Highlighted: None Data

# Comparing the models of amino acid replacements

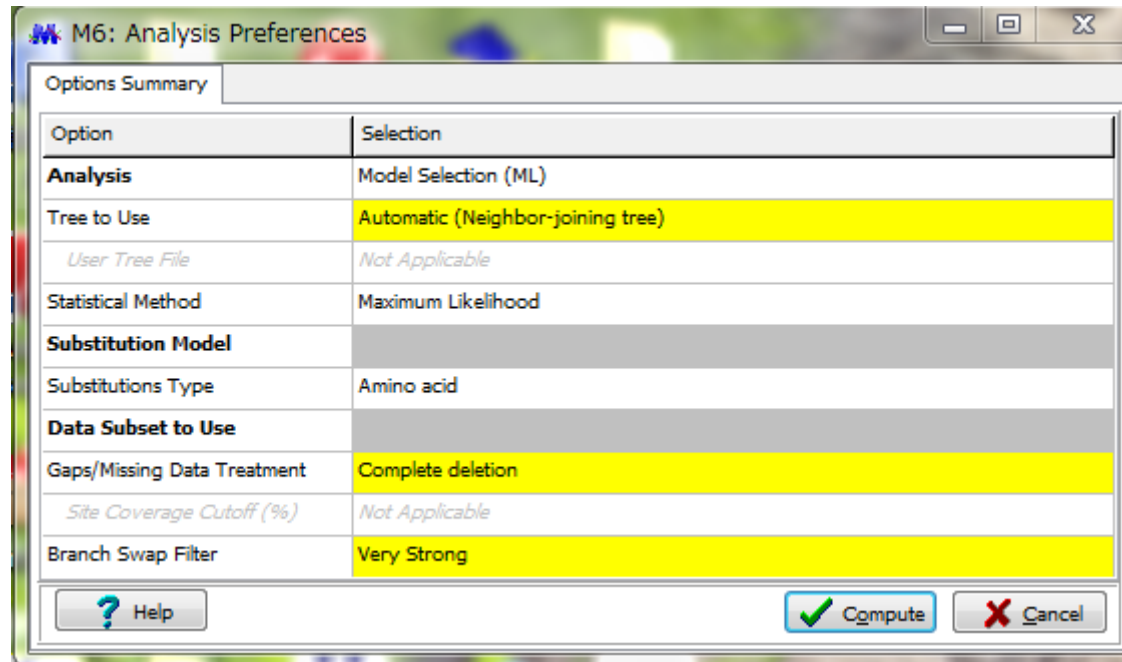
[Models][Find Best DNA/Protein Models (ML)]





# Comparing the models of amino acid replacements

List of options



Option	Selection
<b>Analysis</b>	Model Selection (ML)
Tree to Use	Automatic (Neighbor-joining tree)
<i>User Tree File</i>	<i>Not Applicable</i>
Statistical Method	Maximum Likelihood
<b>Substitution Model</b>	
Substitutions Type	Amino acid
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	<i>Not Applicable</i>
Branch Swap Filter	Very Strong

? Help      ✓ Compute      ✗ Cancel

You can change the options of the items colored yellow.

Here, keep the default values, and click [Compute].

# Comparing the models of amino acid replacements

You will see the progress of the analysis

M6: Progress

Progress

Status / Options

Run Status

Start time	2015/05/11 11:13:47
Status	Testing Dayhoff+G model

Analysis Options

Analysis

Analysis	Model Selection (ML)
Tree to Use	Automatic (Neighbor-joining tree)
Statistical Method	Maximum Likelihood

Substitution Model

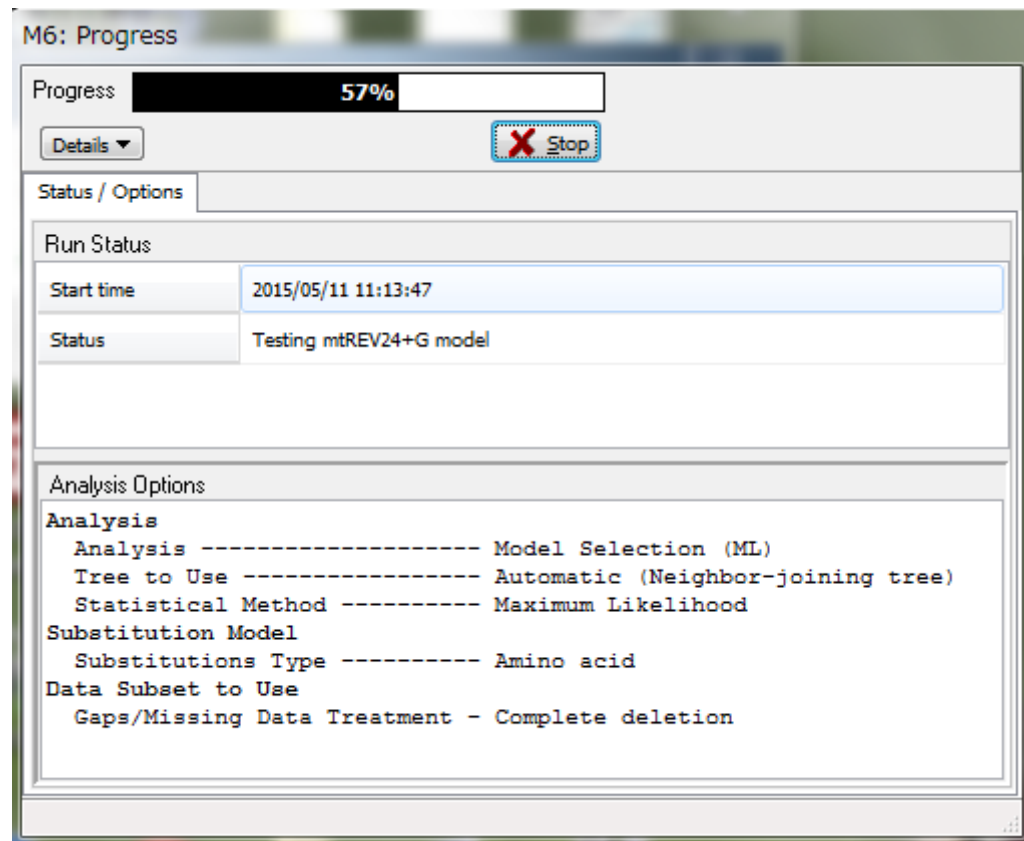
Substitutions Type	Amino acid
--------------------	------------

Data Subset to Use

Gaps/Missing Data Treatment	Complete deletion
-----------------------------	-------------------

# Comparing the models of amino acid replacements

You will see the progress of the analysis



Because various models of aa replacements are applied, it takes some time.

Be patient..

# Comparing the models of amino acid replacements

Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Table. Maximum Likelihood fits of 48 different amino acid substitution models

Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
LG+G+I	43	11960.402	11666.538	-5789.995	0.15	1.14	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
WAG+G+I	43	12046.125	11752.261	-5832.857	0.17	1.37	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
WAG+G	42	12046.744	11759.702	-5837.590	n/a	0.75	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
cpREV+G	42	12068.733	11781.691	-5848.584	n/a	0.72	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
cpREV+G+I	43	12072.005	11778.141	-5845.797	0.16	1.17	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
rtREV+G	42	12072.014	11784.972	-5850.225	n/a	0.72	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
rtREV+G+I	43	12074.609	11780.745	-5847.099	0.15	1.18	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
JTT+G	42	12091.113	11804.071	-5859.774	n/a	0.70	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
JTT+G+I	43	12091.885	11798.021	-5855.737	0.16	1.23	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
LG+G+F	61	12102.453	11685.897	-5781.400	n/a	0.72	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
LG+G+I+F	62	12104.561	11681.194	-5778.030	0.15	1.19	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
rtREV+G+F	61	12149.273	11732.717	-5804.809	n/a	0.72	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
LG+I	42	12151.011	11863.969	-5889.723	0.25	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022

# Comparing the models of amino acid replacements

Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Save Print XL CSV

Tabl	Model	# parameters	BIC	AIC	log likelihood ratio											
	Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
	LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	LG+G+I	43	11960.402	11666.538	-5789.995	0.15	1.14	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	WAG+G+I	43	12046.125	11752.261	-5832.857	0.17	1.37	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
	WAG+G	42	12046.744	11759.702	-5837.590	n/a	0.75	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
	cpREV+G	42	12068.733	11781.691	-5848.584	n/a	0.72	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
	cpREV+G+I	43	12072.005	11778.141	-5845.797	0.16	1.17	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
	rtREV+G	42	12072.014	11784.972	-5850.225	n/a	0.72	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
	rtREV+G+I	43	12074.609	11780.745	-5847.099	0.15	1.18	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
	JTT+G	42	12091.113	11804.071	-5859.774	n/a	0.70	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
	JTT+G+I	43	12091.885	11798.021	-5855.737	0.16	1.23	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
	LG+G+F	61	12102.453	11685.897	-5781.400	n/a	0.72	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
	LG+G+I+F	62	12104.561	11681.194	-5778.030	0.15	1.19	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
	rtREV+G+F	61	12149.273	11732.717	-5804.809	n/a	0.72	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
	LG+I	42	12151.011	11863.969	-5889.723	0.25	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022

# Comparing the models of amino acid replacements

Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Proportion of invariant sites, shape parameter of the gamma distribution ...

Tabl	Model	# parameters	BIC	AIC	log likelihood ratio												
	Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)	
	LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022	
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# Comparing the models of amino acid replacements

Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Proportion of invariant sites, shape parameter of the gamma distribution . . .

Tabl	Model	# parameters	BIC	AIC	log likelihood ratio											
	Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
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	LG+I	42	12151.011	11863.969	-5889.723	0.25	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022

# Comparing the models of amino acid replacements

Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Proportion of invariant sites, shape parameter of the gamma distribution ...

Tabl	Model	# parameters	BIC	AIC	log likelihood ratio											
	Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
Best model	LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	LG+G+I	43	11960.402	11666.538	-5789.995	0.15	1.14	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	WAG+G+I	43	12046.125	11752.261	-5832.857	0.17	1.37	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
	WAG+G	● Amino acid replacement matrix: LG model														
	cpREV+G	42	12068.733	11781.691	-5848.584	n/a	0.72	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
	cpREV+G+I	43	12072.005	11778.141	-5845.797	0.16	1.17	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
	rtREV+G	42	12072.014	11784.972	-5850.225	n/a	0.72	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
	rtREV+G+I	43	12074.609	11780.745	-5847.099	0.15	1.18	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
	JTT+G	42	12091.113	11804.071	-5859.774	n/a	0.70	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
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	LG+G+I+F	62	12104.561	11681.194	-5778.030	0.15	1.19	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
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	LG+I	42	12151.011	11863.969	-5889.723	0.25	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022



# Comparing the models of amino acid replacements


Once finished, you will see the list of models ranked with BIC.

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Proportion of invariant sites, shape parameter of the gamma distribution . . .

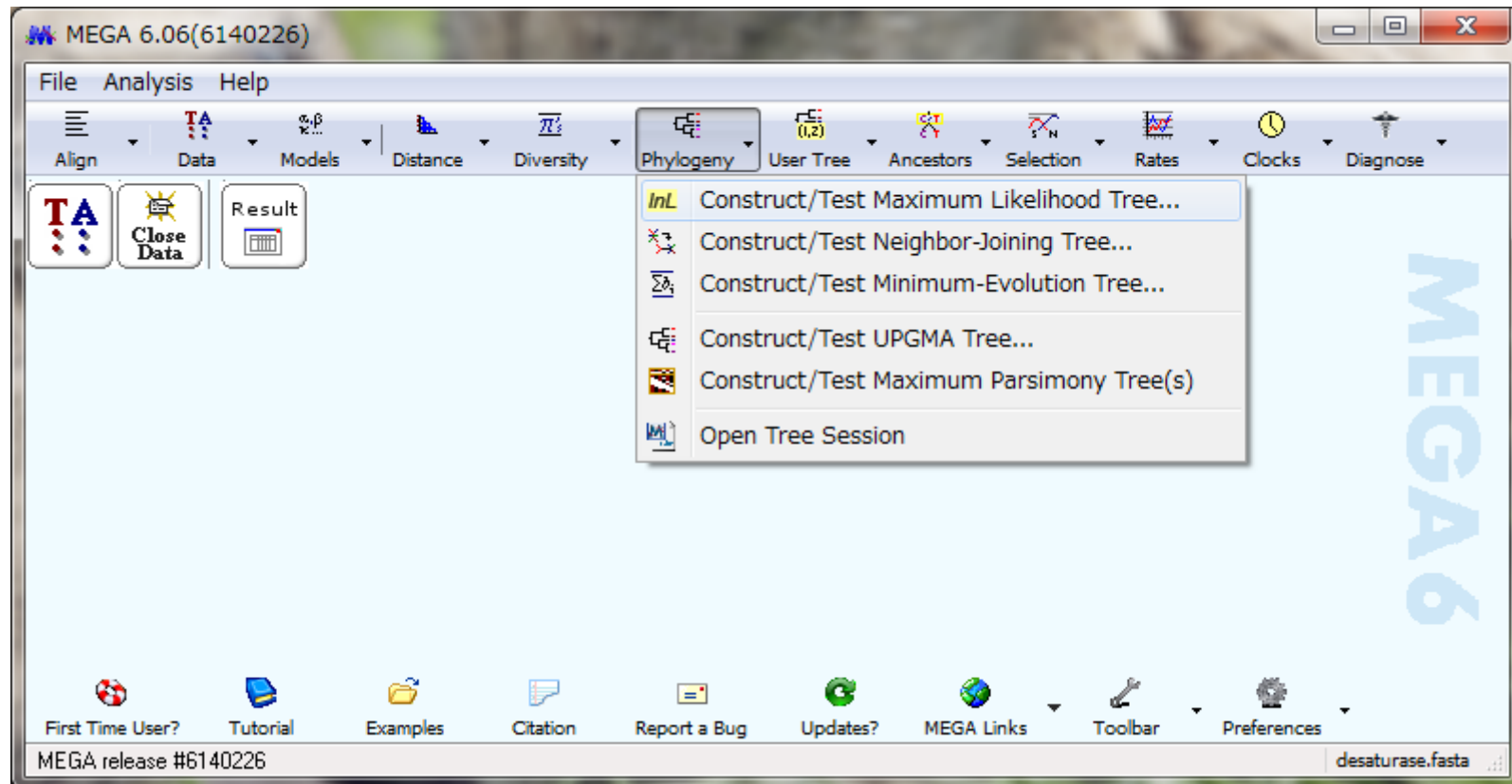
Tabl	Model	# parameters	BIC	AIC	log likelihood ratio											
	Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
Best model	LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	LG+G+I	43	11960.402	11666.538	-5789.995	0.15	1.14	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
	WAG+G+I	43	12046.125	11752.261	-5832.857	0.17	1.37	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
	WAG+G	● Amino acid replacement matrix: LG model														
	cpREV+G	● Among site rate variation: gamma distribution														
	cpREV+G+I	43	12072.005	11778.141	-5845.797	0.16	1.17	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
	rtREV+G	42	12072.014	11784.972	-5850.225	n/a	0.72	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
	rtREV+G+I	43	12074.609	11780.745	-5847.099	0.15	1.18	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027
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	JTT+G+I	43	12091.885	11798.021	-5855.737	0.16	1.23	0.077	0.051	0.043	0.051	0.020	0.041	0.062	0.075	0.023
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	rtREV+G+F	61	12149.273	11732.717	-5804.809	n/a	0.72	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
	LG+I	42	12151.011	11863.969	-5889.723	0.25	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022

A dark blue circle is centered on the page. Inside the circle, the text "Maximum likelihood inference" is written in white, sans-serif font, arranged in three lines.

Maximum  
likelihood  
inference

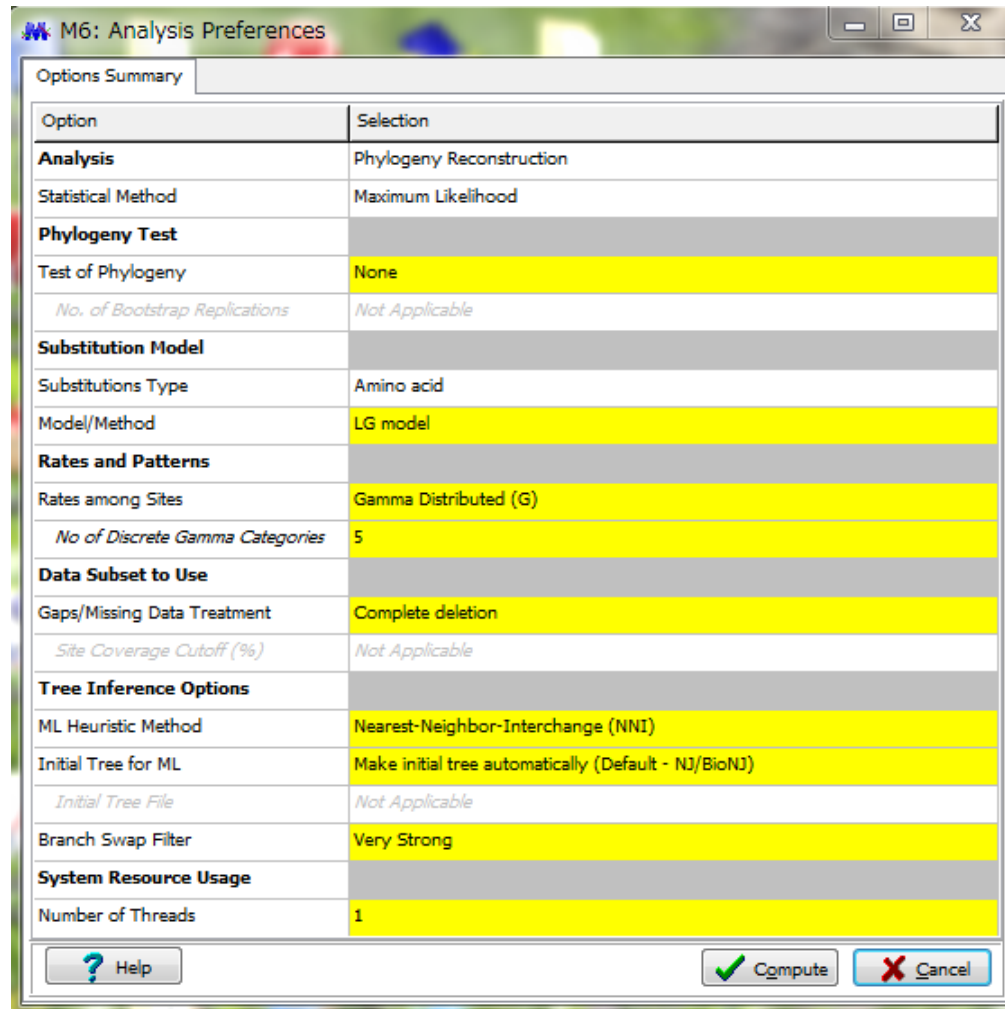
# Constructing a phylogenetic tree by the best model

[Phylogeny][Construct/Test Maximum Likelihood Tree]



# Constructing a phylogenetic tree by the best model

List of options



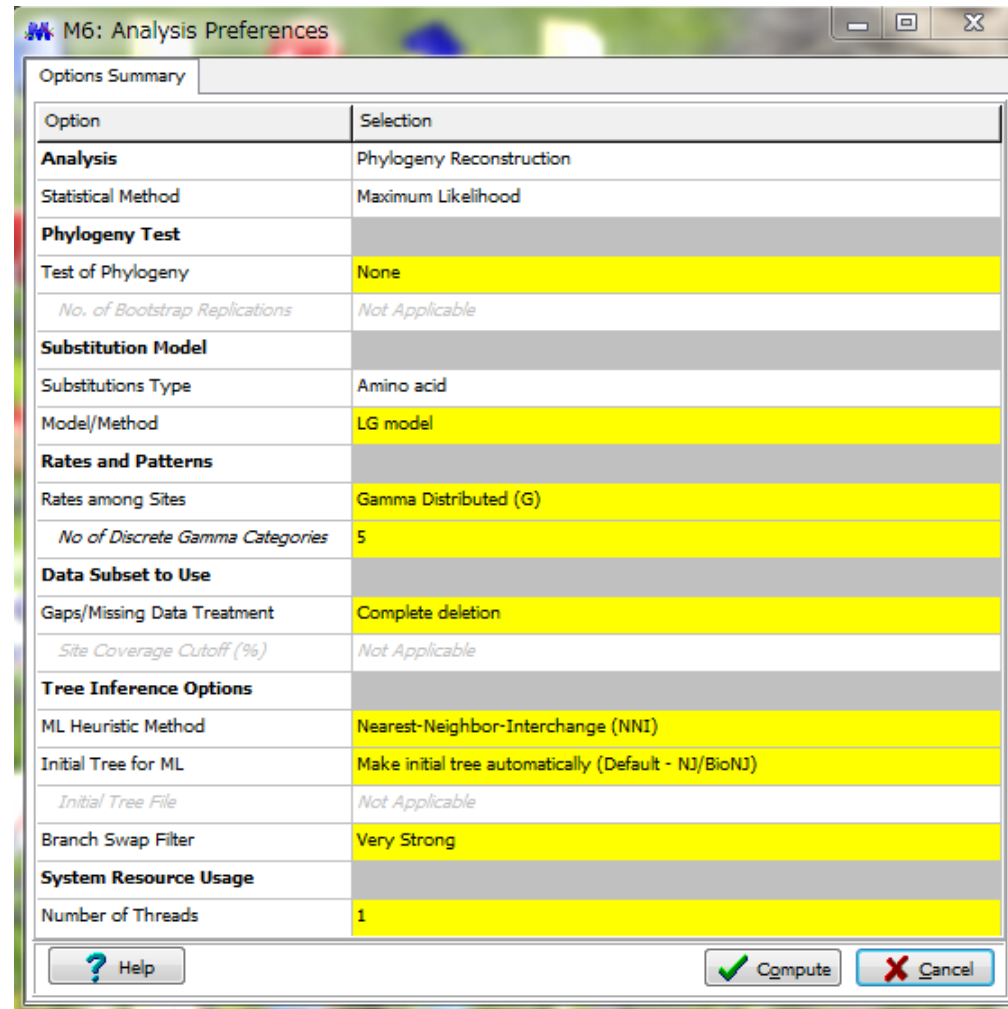
The screenshot shows a software window titled "M6: Analysis Preferences". It contains a table of options and their selected values. The options are grouped into sections: Analysis, Phylogeny Test, Substitution Model, Rates and Patterns, Data Subset to Use, Tree Inference Options, and System Resource Usage. The selected values are: Phylogeny Reconstruction, Maximum Likelihood, None, Not Applicable, Amino acid, LG model, Gamma Distributed (G), 5, Complete deletion, Not Applicable, Nearest-Neighbor-Interchange (NNI), Make initial tree automatically (Default - NJ/BioNJ), Not Applicable, Very Strong, and 1. At the bottom, there are buttons for Help, Compute, and Cancel.

Option	Selection
<b>Analysis</b>	Phylogeny Reconstruction
Statistical Method	Maximum Likelihood
<b>Phylogeny Test</b>	
Test of Phylogeny	None
<i>No. of Bootstrap Replications</i>	<i>Not Applicable</i>
<b>Substitution Model</b>	
Substitutions Type	Amino acid
Model/Method	LG model
<b>Rates and Patterns</b>	
Rates among Sites	Gamma Distributed (G)
<i>No of Discrete Gamma Categories</i>	5
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	<i>Not Applicable</i>
<b>Tree Inference Options</b>	
ML Heuristic Method	Nearest-Neighbor-Interchange (NNI)
Initial Tree for ML	Make initial tree automatically (Default - NJ/BioNJ)
<i>Initial Tree File</i>	<i>Not Applicable</i>
Branch Swap Filter	Very Strong
<b>System Resource Usage</b>	
Number of Threads	1

? Help      ✓ Compute      ✗ Cancel

# Constructing a phylogenetic tree by the best model

List of options



The screenshot shows a dialog box titled "M6: Analysis Preferences" with a tab labeled "Options Summary". The dialog contains a table with two columns: "Option" and "Selection". The options are grouped into sections: Analysis, Phylogeny Test, Substitution Model, Rates and Patterns, Data Subset to Use, Tree Inference Options, and System Resource Usage. The selected values are: Phylogeny Reconstruction, Maximum Likelihood, None, Not Applicable, Amino acid, LG model, Gamma Distributed (G), 5, Complete deletion, Not Applicable, Nearest-Neighbor-Interchange (NNI), Make initial tree automatically (Default - NJ/BioNJ), Not Applicable, Very Strong, and 1. At the bottom, there are buttons for Help, Compute, and Cancel.

Option	Selection
<b>Analysis</b>	Phylogeny Reconstruction
Statistical Method	Maximum Likelihood
<b>Phylogeny Test</b>	
Test of Phylogeny	None
<i>No. of Bootstrap Replications</i>	<i>Not Applicable</i>
<b>Substitution Model</b>	
Substitutions Type	Amino acid
Model/Method	LG model
<b>Rates and Patterns</b>	
Rates among Sites	Gamma Distributed (G)
<i>No of Discrete Gamma Categories</i>	5
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	<i>Not Applicable</i>
<b>Tree Inference Options</b>	
ML Heuristic Method	Nearest-Neighbor-Interchange (NNI)
Initial Tree for ML	Make initial tree automatically (Default - NJ/BioNJ)
<i>Initial Tree File</i>	<i>Not Applicable</i>
Branch Swap Filter	Very Strong
<b>System Resource Usage</b>	
Number of Threads	1

? Help    ✓ Compute    ✗ Cancel

bootstrap (Y/N)

aa replacement matrix

Variable rate among sites

# Constructing a phylogenetic tree by the best model

List of options

M6: Analysis Preferences

Options Summary

Option	Selection
<b>Analysis</b>	Phylogeny Reconstruction
Statistical Method	Maximum Likelihood
<b>Phylogeny Test</b>	
Test of Phylogeny	None
<i>No. of Bootstrap Replications</i>	<i>Not Applicable</i>
<b>Substitution Model</b>	
Substitutions Type	Amino acid
Model/Method	LG model
<b>Rates and Patterns</b>	
Rates among Sites	Gamma Distributed (G)
<i>No of Discrete Gamma Categories</i>	5
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	<i>Not Applicable</i>
<b>Tree Inference Options</b>	
ML Heuristic Method	Nearest-Neighbor-Interchange (NNI)
Initial Tree for ML	Make initial tree automatically (Default - NJ/BioNJ)
<i>Initial Tree File</i>	<i>Not Applicable</i>
Branch Swap Filter	Very Strong
<b>System Resource Usage</b>	
Number of Threads	1

? Help

✓ Compute

✗ Cancel

bootstrap (Y/N)

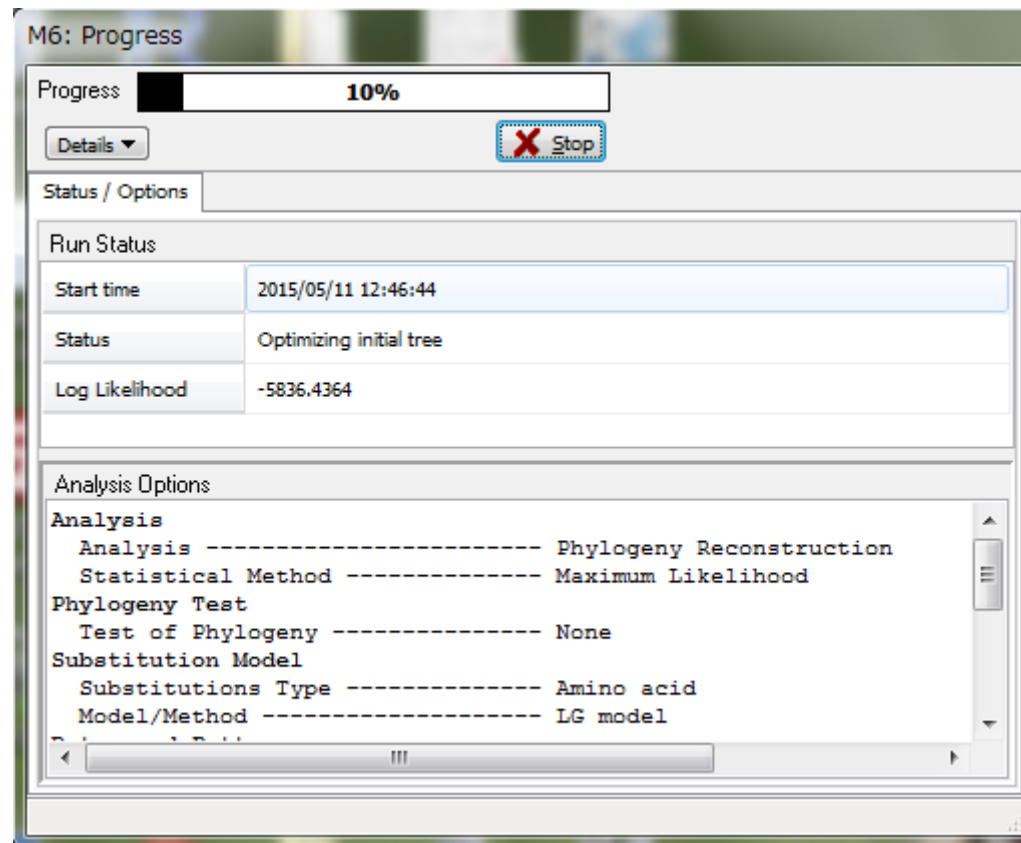
aa replacement matrix

Variable rate among sites

If ready, click Compute].

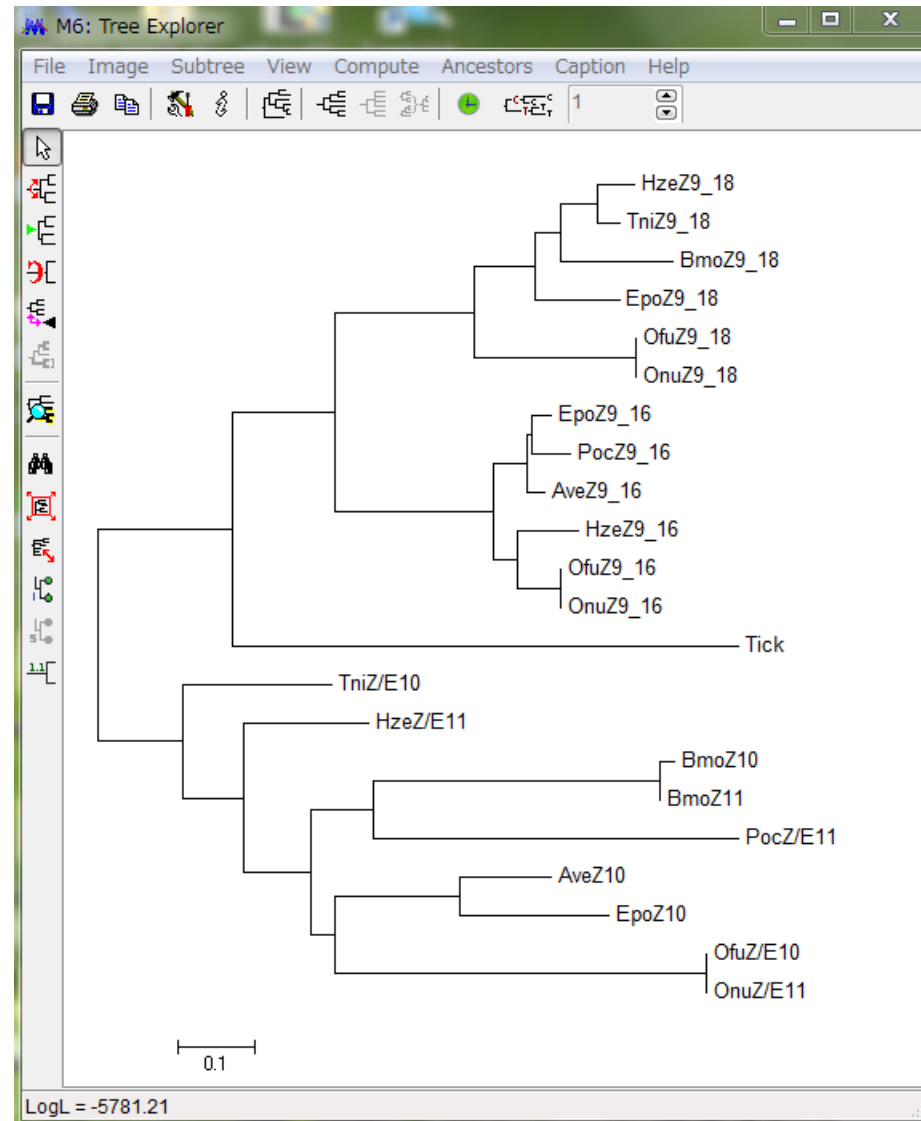
# Constructing a phylogenetic tree by the best model

Computation starts ...



# Constructing a phylogenetic tree by the best model

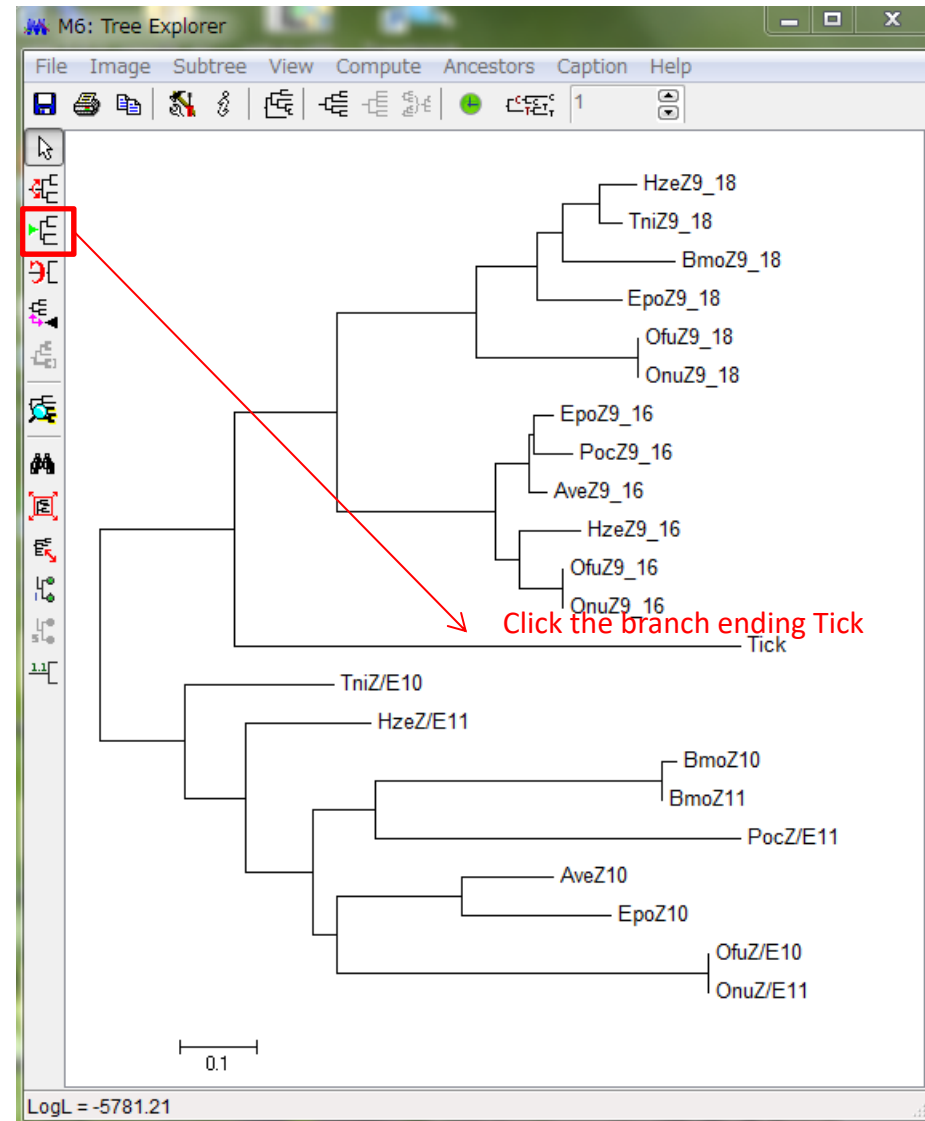
Once the phylogenetic tree is constructed, you will see the figure.





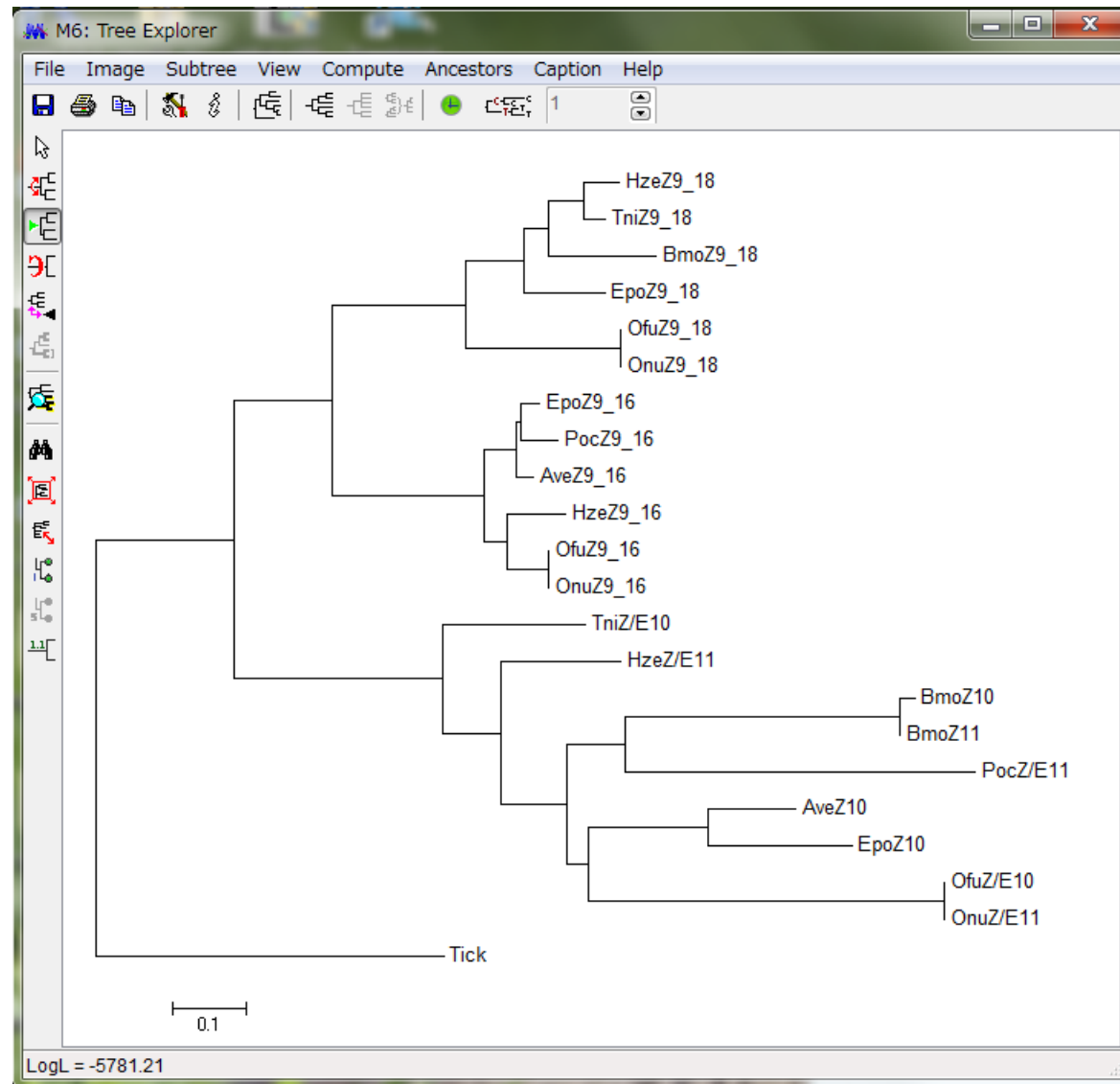
# Constructing a phylogenetic tree by the best model

Click  and specify Tick as an outgroup



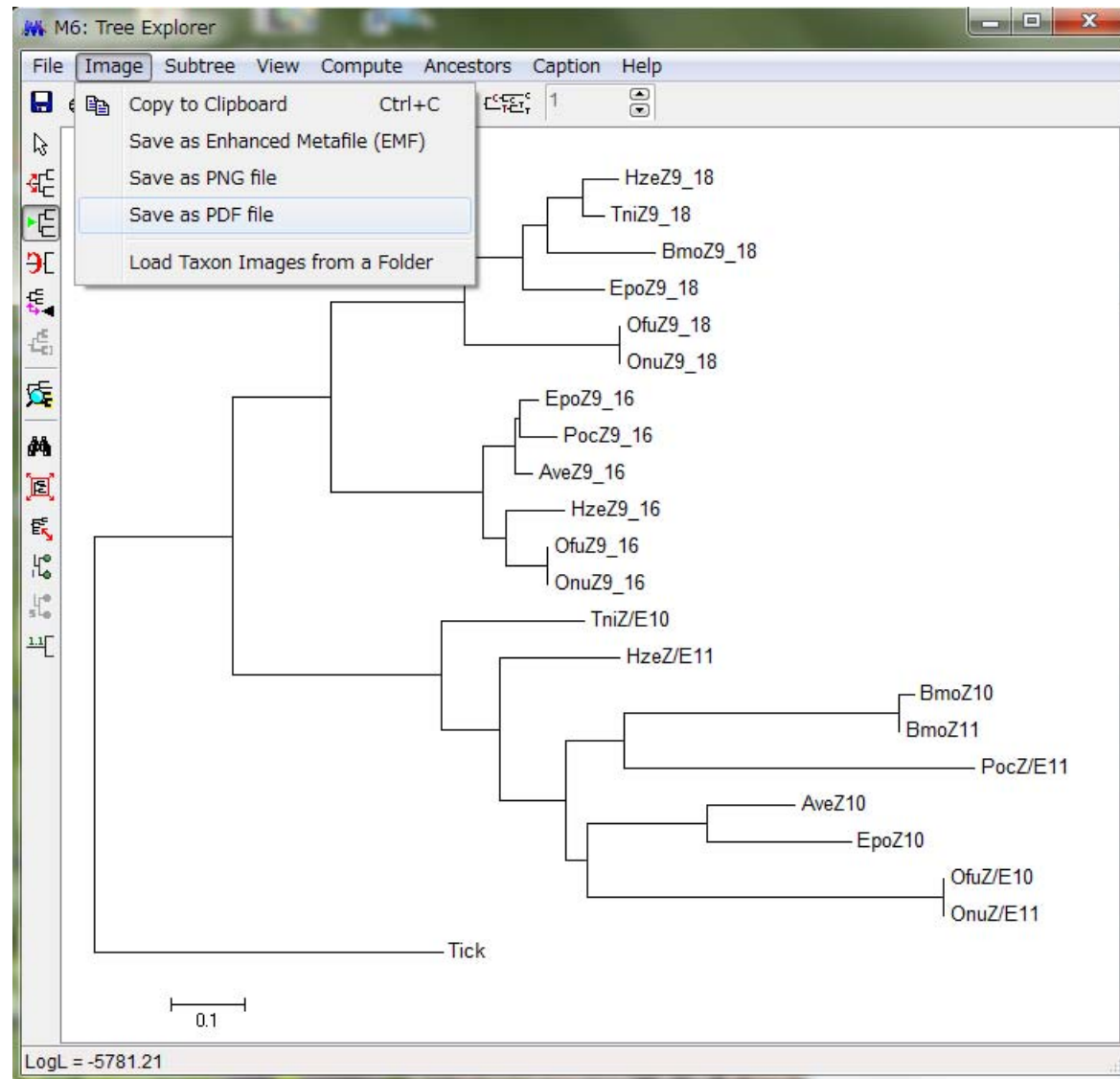
# Constructing a phylogenetic tree by the best model

Rooted tree with Tick as an outgroup

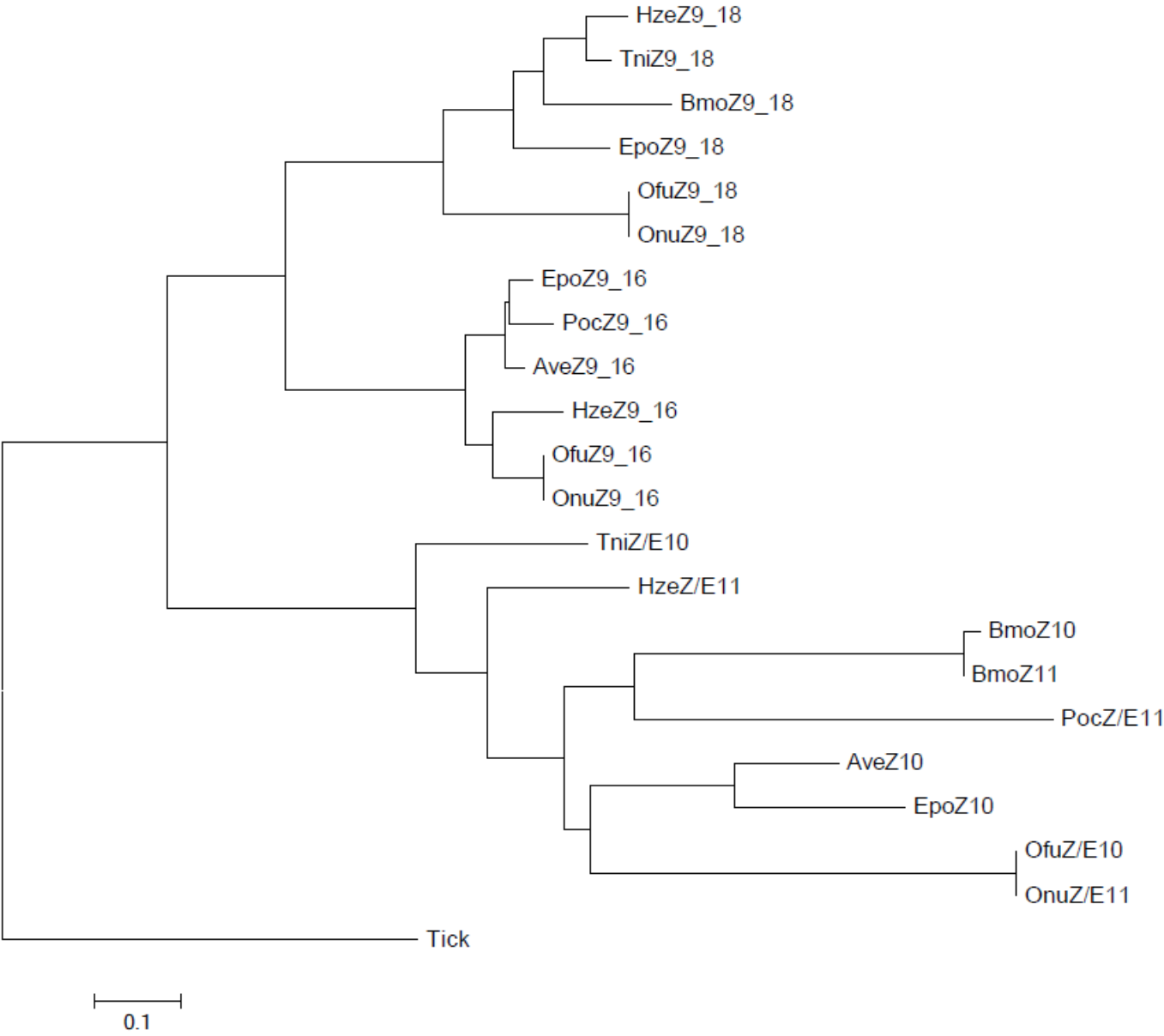


# Save the figure as a file

[Image][Save as PDF file] (choice of format: clipboard, emf, png, pdf ...)

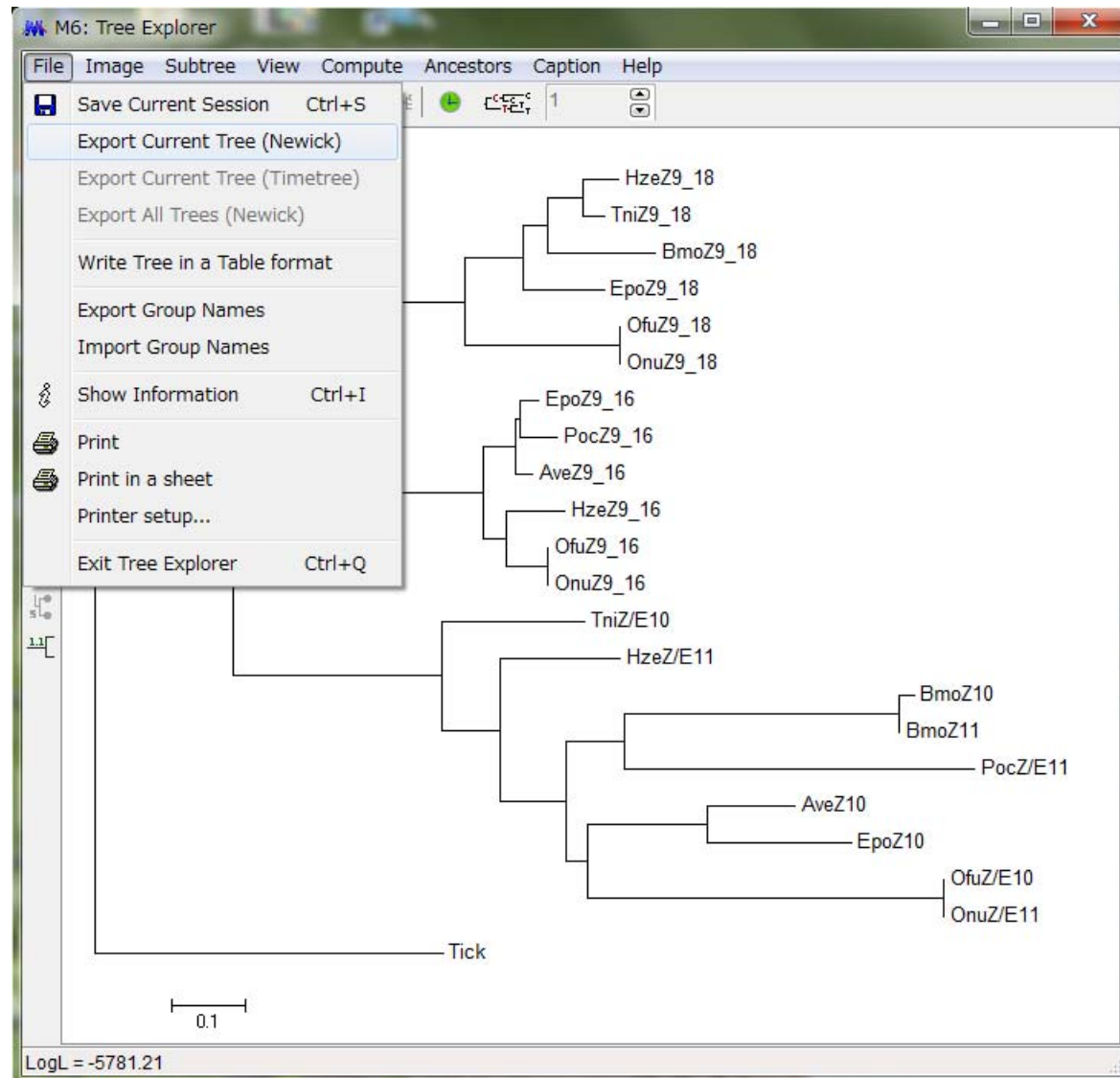


# Saved image file




# Save the tree information as a newick file

[File][Export Current Tree (Newick)]



## Newick file

```
(((((('HzeZ9_18':0.04744891,'TniZ9_18':0.02873954):0.04876410,'BmoZ9_18':0.14668082):0.03363353,'EpoZ9_18':0.10998264):0.08067921,('OfuZ9_18':0.00000000,'OnuZ9_18':0.00000000):0.21168310):0.18028008,(((('EpoZ9_16':0.02681811,'PocZ9_16':0.05093039):0.00494556,'AveZ9_16':0.02186046):0.04527333,('HzeZ9_16':0.08043957,('OfuZ9_16':0.00000000,'OnuZ9_16':0.00000000):0.05777886):0.03121448):0.20549621):0.13563580,(TniZ/E10:0.19565117,(HzeZ/E11:0.16265867,(((BmoZ10:0.01952739,BmoZ11:0.00000000):0.37509960,PocZ/E11:0.47698127):0.08048141,((AveZ10:0.11881414,EpoZ10:0.19493515):0.16379172,(OfuZ/E10:0.00000000,OnuZ/E11:0.00000000):0.48603868):0.03036878):0.08837050):0.08074866):0.28506824):0.18749402,Tick:0.47416490);
```



**More thought  
on the rate  
heterogeneity  
among sites**

# Heterogeneity/homogeneity among sites

MEGA Caption Expert: Find Best-Fit Substitution Model (ML)

File Edit View Help

Table. Maximum Likelihood fits of 48 different amino acid substitution models

Model	Parameters	BIC	AICc	lnL	(+I)	(+G)	f(A)	f(R)	f(N)	f(D)	f(C)	f(Q)	f(E)	f(G)	f(H)
LG+G	42	11957.554	11670.512	-5792.995	n/a	0.70	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
LG+G+I	43	11960.402	11666.538	-5789.995	0.15	1.14	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
WAG+G+I	43	12046.125	11752.261	-5832.857	0.17	1.37	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
WAG+G	42	12046.744	11759.702	-5837.590	n/a	0.75	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
cpREV+G	42	12068.733	11781.691	-5848.584	n/a	0.72	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
cpREV+G+I	43	12072.005	11778.141	-5845.797	0.16	1.17	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025

Best model

...

JTT+I+F	61	12399.213	11982.657	-5929.780	0.25	n/a	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
Dayhoff+I	42	12408.718	12121.676	-6018.576	0.25	n/a	0.087	0.041	0.040	0.047	0.034	0.038	0.050	0.089	0.034
LG	41	12442.188	12161.968	-6039.735	n/a	n/a	0.079	0.056	0.042	0.053	0.013	0.041	0.072	0.057	0.022
Dayhoff+I+F	61	12445.342	12028.786	-5952.844	0.25	n/a	0.083	0.045	0.045	0.048	0.010	0.019	0.029	0.060	0.050
WAG	41	12506.129	12225.909	-6071.705	n/a	n/a	0.087	0.044	0.039	0.057	0.019	0.037	0.058	0.083	0.024
cpREV	41	12526.120	12245.900	-6081.701	n/a	n/a	0.076	0.062	0.041	0.037	0.009	0.038	0.050	0.084	0.025
rtREV	41	12553.704	12273.484	-6095.493	n/a	n/a	0.065	0.045	0.038	0.042	0.011	0.061	0.061	0.064	0.027

...

Assuming uniform rate among sites



# Effect of ignoring rate heterogeneity among sites

[Phylogeny][Construct/Test Maximum Likelihood Tree]  
-> [Rates among Sites]: Choose “Uniform rates”

M6: Analysis Preferences

Options Summary

Option	Selection
<b>Analysis</b>	Phylogeny Reconstruction
Statistical Method	Maximum Likelihood
<b>Phylogeny Test</b>	
Test of Phylogeny	None
<i>No. of Bootstrap Replications</i>	Not Applicable
<b>Substitution Model</b>	
Substitutions Type	Amino acid
Model/Method	LG model
<b>Rates and Patterns</b>	
Rates among Sites	Gamma Distributed (G)
<i>No of Discrete Gamma Categories</i>	5
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	Not Applicable
<b>Tree Inference Options</b>	
ML Heuristic Method	Nearest-Neighbor-Interchange (NNI)
Initial Tree for ML	Make initial tree automatically (Default - NJ/BioNJ)
<i>Initial Tree File</i>	Not Applicable
Branch Swap Filter	Very Strong
<b>System Resource Usage</b>	
Number of Threads	1

? Help    ✓ Compute    ✗ Cancel



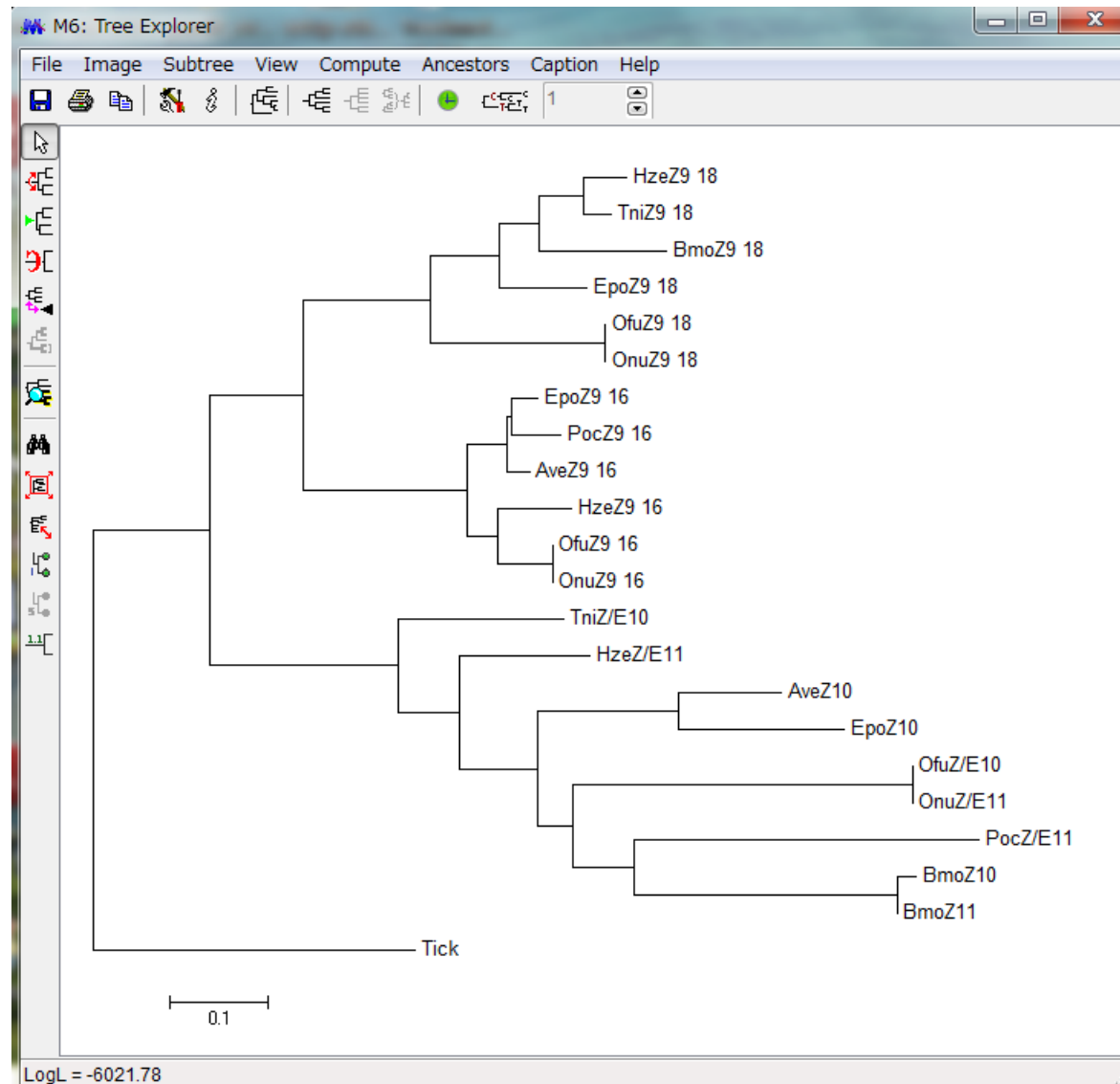
M6: Analysis Preferences

Options Summary

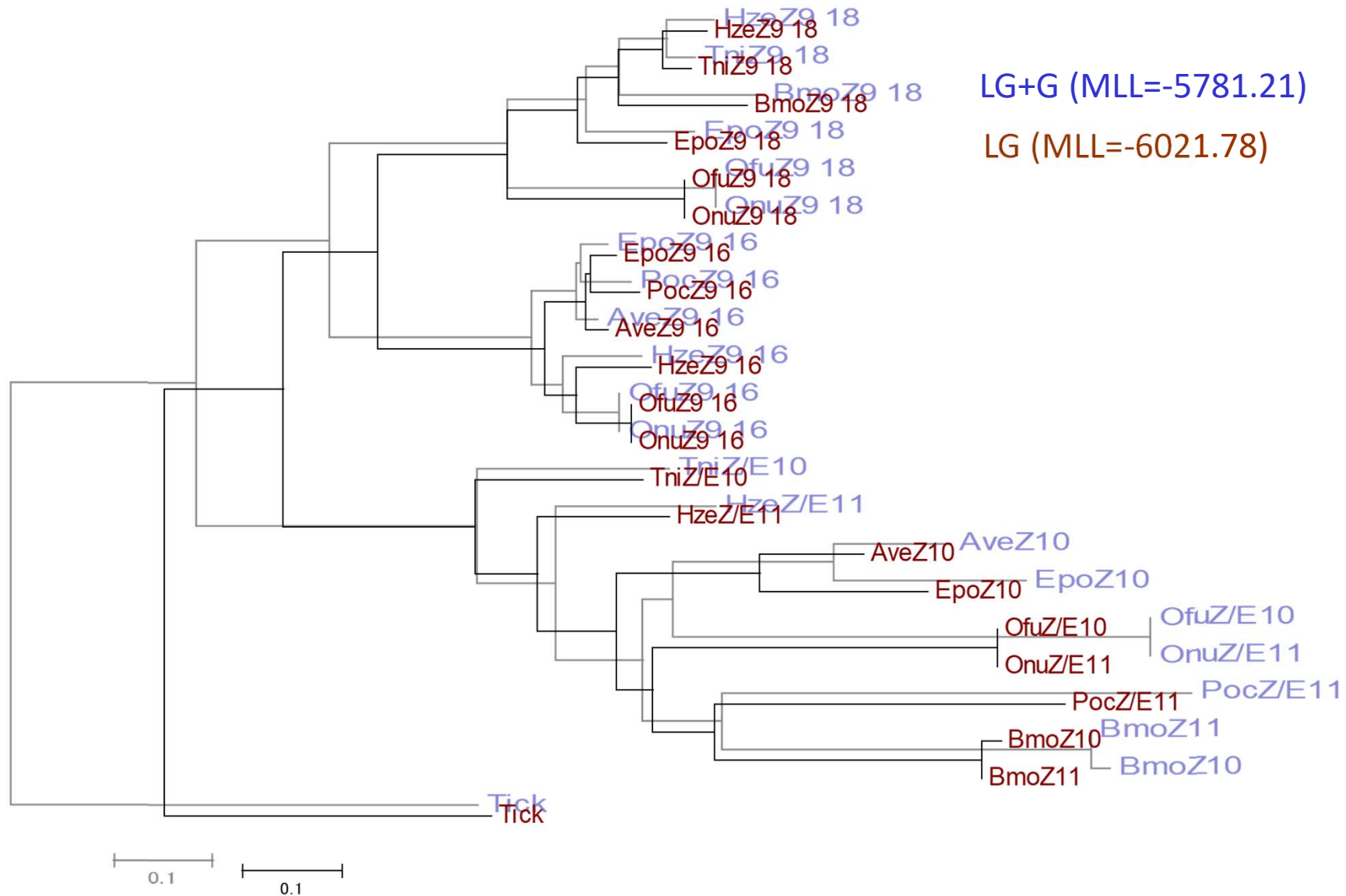
Option	Selection
<b>Analysis</b>	Phylogeny Reconstruction
Statistical Method	Maximum Likelihood
<b>Phylogeny Test</b>	
Test of Phylogeny	None
<i>No. of Bootstrap Replications</i>	Not Applicable
<b>Substitution Model</b>	
Substitutions Type	Amino acid
Model/Method	LG model
<b>Rates and Patterns</b>	
Rates among Sites	Has Invariant sites (I)
<i>No of Discrete Gamma Categories</i>	Uniform rates
<b>Data Subset to Use</b>	
Gaps/Missing Data Treatment	Complete deletion
<i>Site Coverage Cutoff (%)</i>	Not Applicable
<b>Tree Inference Options</b>	
ML Heuristic Method	Nearest-Neighbor-Interchange (NNI)
Initial Tree for ML	Make initial tree automatically (Default - NJ/BioNJ)
<i>Initial Tree File</i>	Not Applicable
Branch Swap Filter	Very Strong
<b>System Resource Usage</b>	
Number of Threads	1


? Help    ✓ Compute    ✗ Cancel

# Effect of ignoring rate heterogeneity among sites



# Effect of ignoring rate heterogeneity among sites



A dark blue circle is centered on the slide, containing the text '[coffee break] Maximum likelihood inference'.

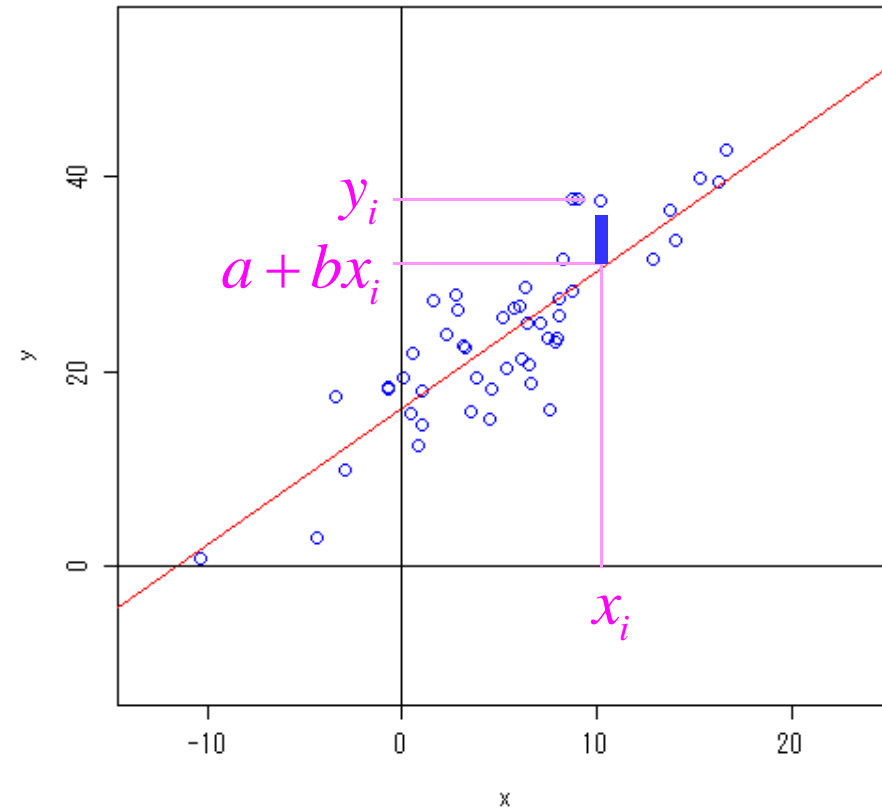
[coffee break]  
Maximum  
likelihood  
inference

# Likelihood and maximum likelihood (ML) procedure

- Likelihood describes the probability of observing the data by a statistical model.
- Likelihood function: explicit representation of the likelihood in terms of parameters.
- Log likelihood: log of likelihood
- ML procedure: the method of estimating the parameters by maximizing the log likelihood value
  - MLE: the estimate by ML procedure
  - The variance of the MLE is obtained by the inverse of Fisher information quantity (minus the second derivative (Hessian) of the log likelihood function).

## ML procedure for Regression analysis

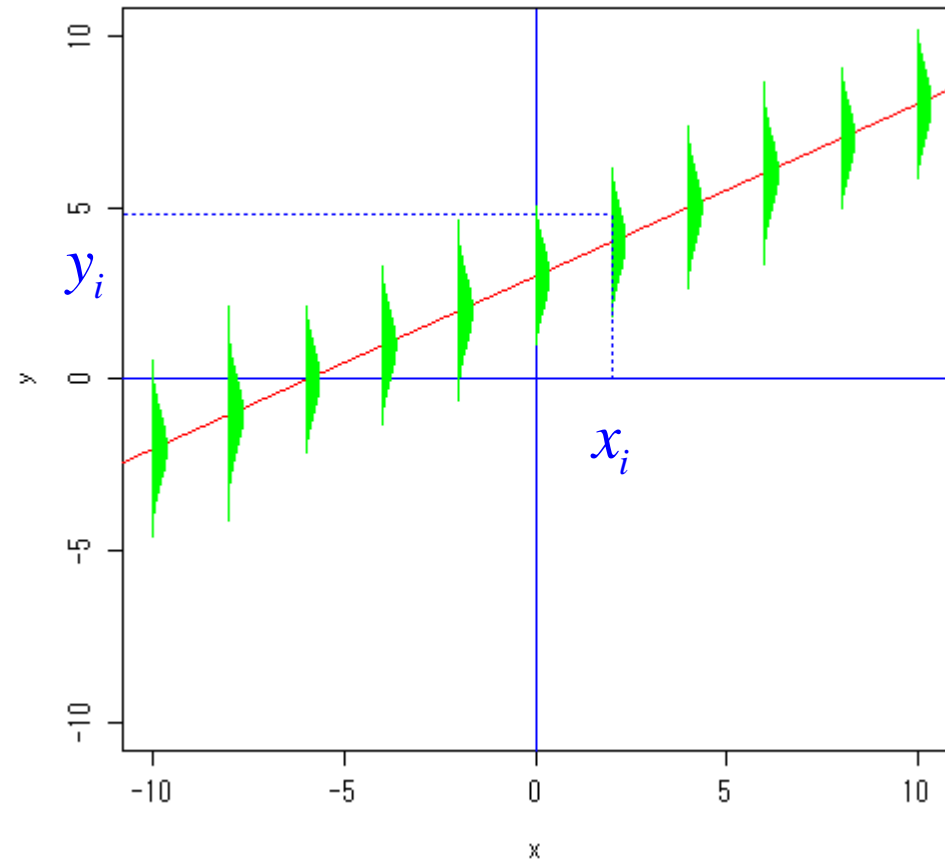
$$y_i = a + bx_i + \varepsilon_i \quad i = 1, \dots, n$$



# ML procedure for Regression analysis

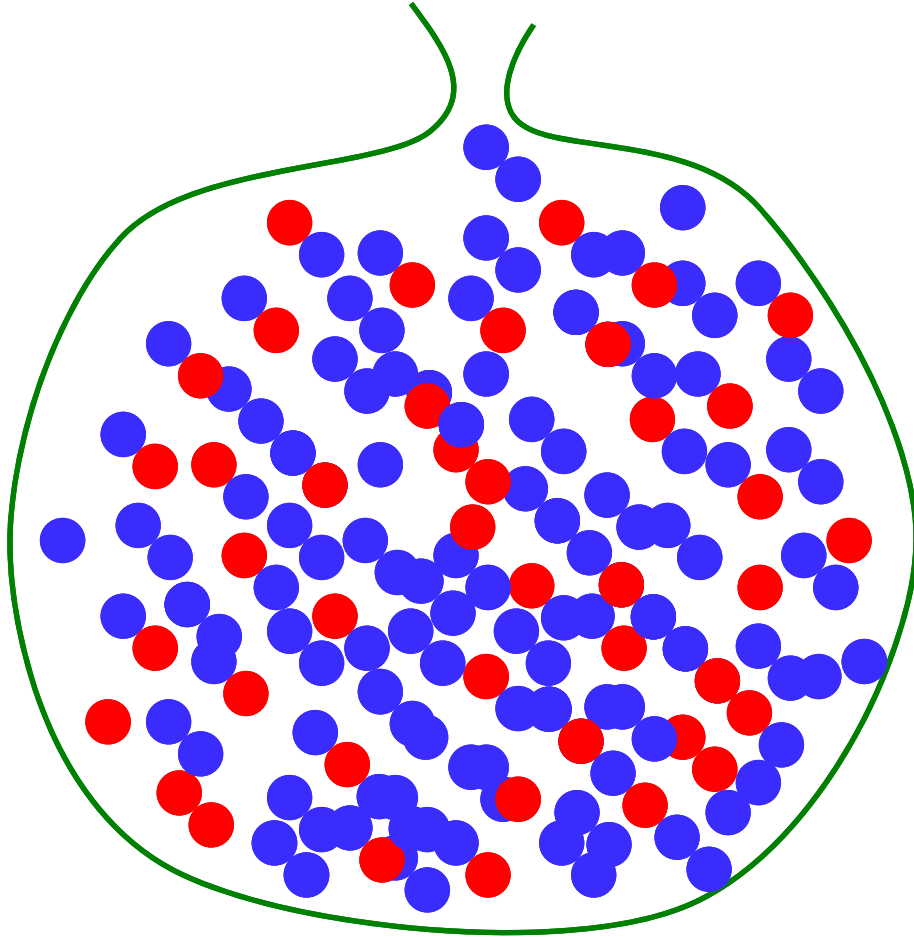
Statistical model and the likelihood

$$\begin{aligned} & L(a, b, s^2 \mid y_1, \dots, y_n) \\ &= p(y_1) \cdots p(y_n) \\ &= \left( \frac{1}{\sqrt{2\pi s^2}} \right)^n \exp \left[ - \sum_{i=1}^n \frac{(y_i - (a + bx_i))^2}{2s^2} \right] \end{aligned}$$



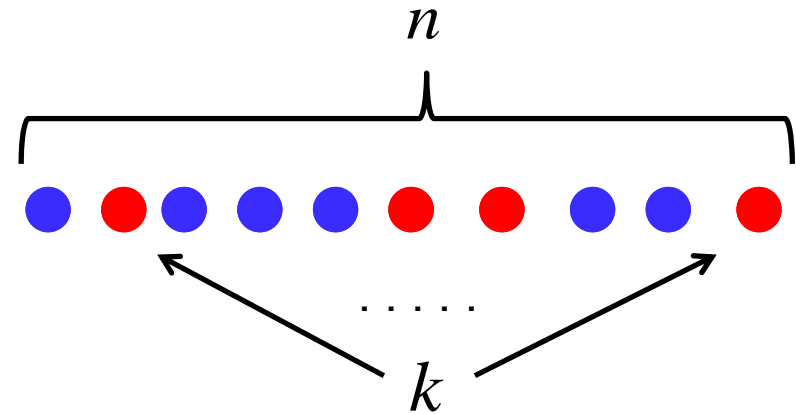
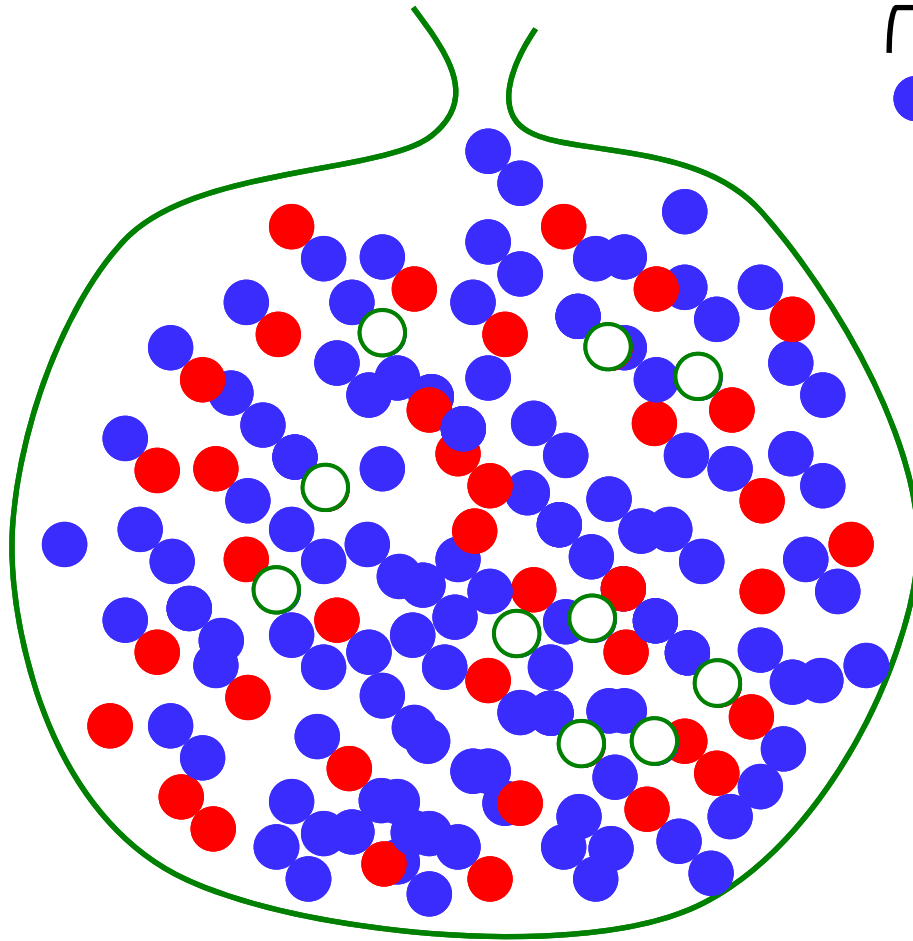
The least squares estimate is the ML estimate assuming normal distribution of error terms.

ML procedure for estimation of the probability





ML procedure for estimation of the probability



$$L = p^k (1 - p)^{n-k}$$

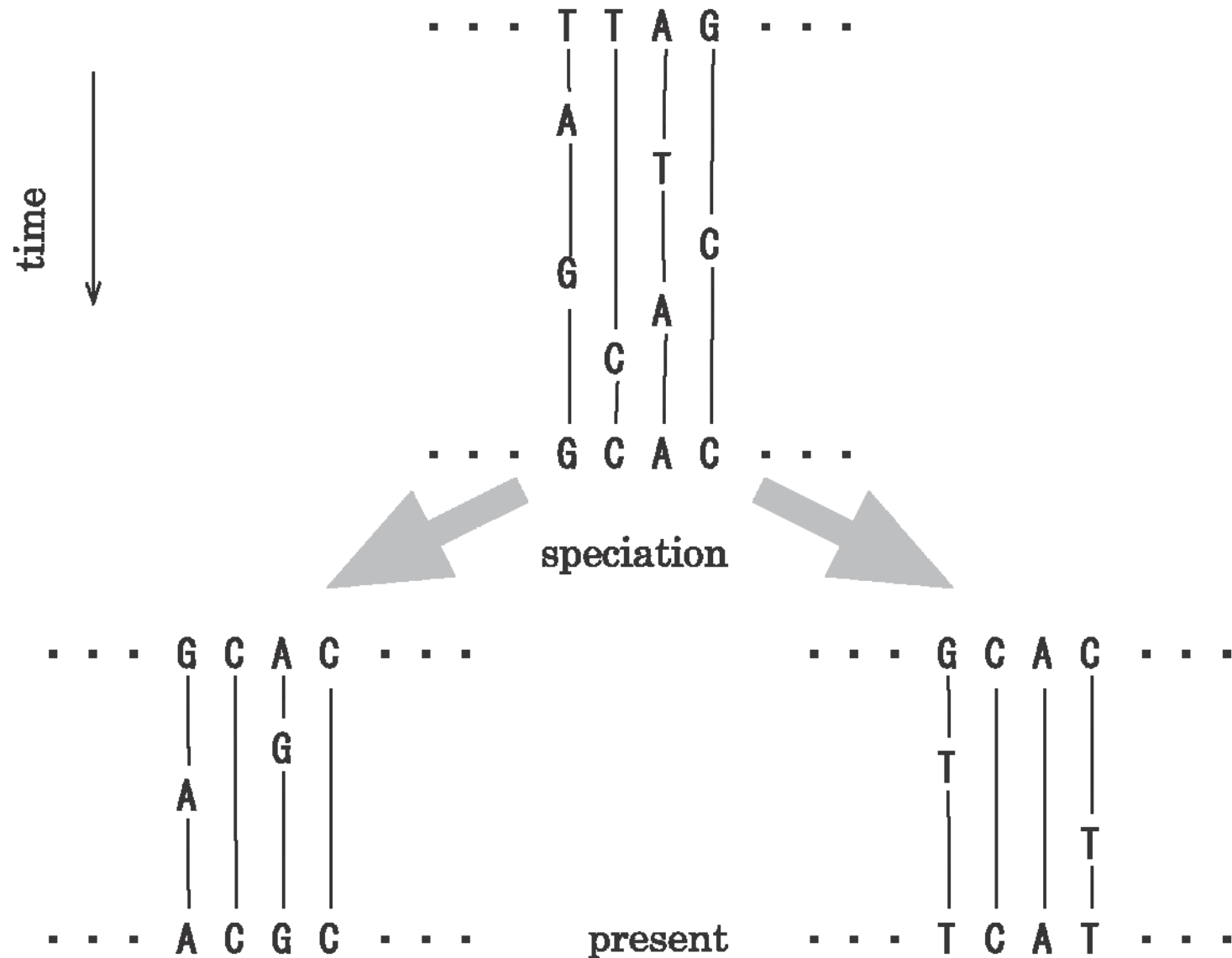
$$\lambda = \log L$$

$$= k \log p + (n - k) \log(1 - p)$$

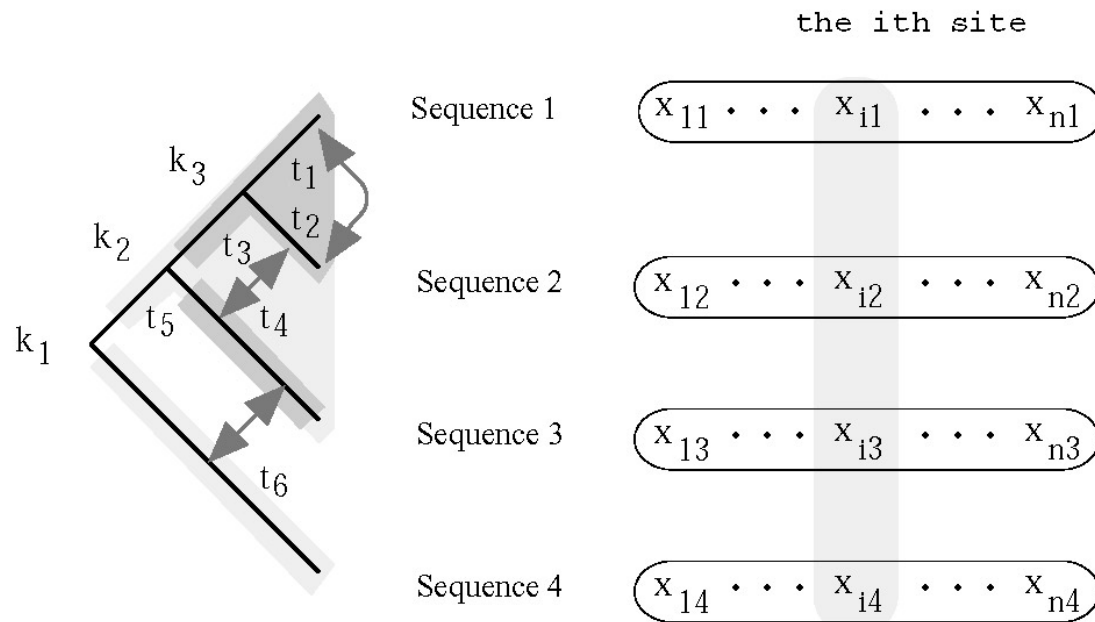
$$\frac{d\lambda}{dp} = \frac{k}{p} + \frac{n-k}{p-1} = 0$$

$$\hat{p} = \frac{k}{n}$$

# Statistical model of molecular evolution



# Likelihood of a site (alignment column)



$$f(x_{i1}, \dots, x_{i4} | \lambda, t_1, \dots, t_6) = \sum_{k_1} \pi_{k_1} p_{k_1, x_{i4}}(t_6) \left( \sum_{k_2} p_{k_1, k_2}(t_5) \left( p_{k_2, x_{i3}}(t_4) \left( \sum_{k_3} p_{k_2, k_3}(t_3) (p_{k_3, x_{i2}}(t_2) p_{k_3, x_{i1}}(t_1)) \right) \right) \right)$$

$$l(\boldsymbol{\theta} | \mathbf{X}, T) = \sum_{h=1}^n \log f(\mathbf{X}_h | \boldsymbol{\theta}, T) = \sum_{h=1}^n \log \left[ \sum_{Z_{i_0}} \pi_{Z_{i_0}} \prod_{j \in \text{node}(T) \setminus i_0} \sum_{Z_j} P_{Z_{\text{anc}(j)} Z_j} (t_{\text{anc}(j), j}) \right]$$

$$\mathbf{P}(t) = \exp(t\mathbf{R}) = \exp(\mathbf{R}_0)$$

branch lengths

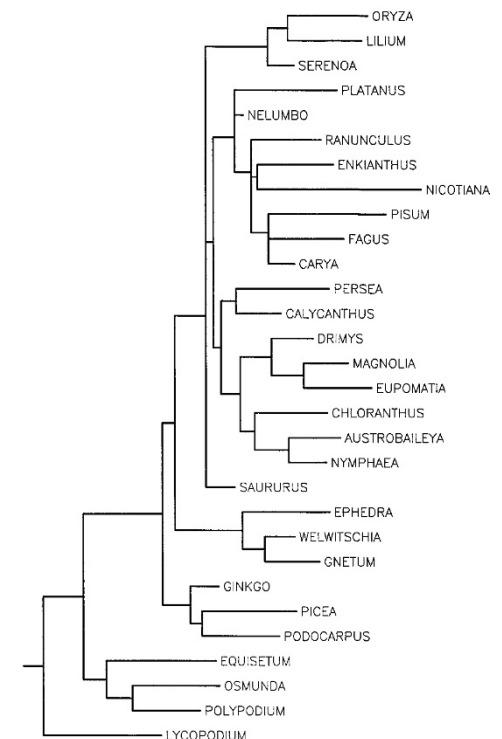
# Likelihood of sequences

Species 1	$X_{11}$	$\dots$	$X_{1q}$	$\dots$	$X_{1n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Species p	$X_{p1}$	$\dots$	$X_{pq}$	$\dots$	$X_{pn}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Species s	$X_{s1}$	$\dots$	$X_{sq}$	$\dots$	$X_{sn}$

$$l(\mathbf{t}, \mathbf{r} | \mathbf{X}) = \log f(\mathbf{X}_1 | \mathbf{t}, \mathbf{r}) + \dots + \log f(\mathbf{X}_q | \mathbf{t}, \mathbf{r}) + \dots + \log f(\mathbf{X}_n | \mathbf{t}, \mathbf{r})$$

$$\mathbf{P}(t) = \exp(t\mathbf{R}) = \exp(\mathbf{tr}\mathbf{R}_0)$$

branch lengths



# Numerical optimization package makes ML easy

An example function

```
g <- function(x)
  { - exp(-x) * x^2*(x+2)^3 }

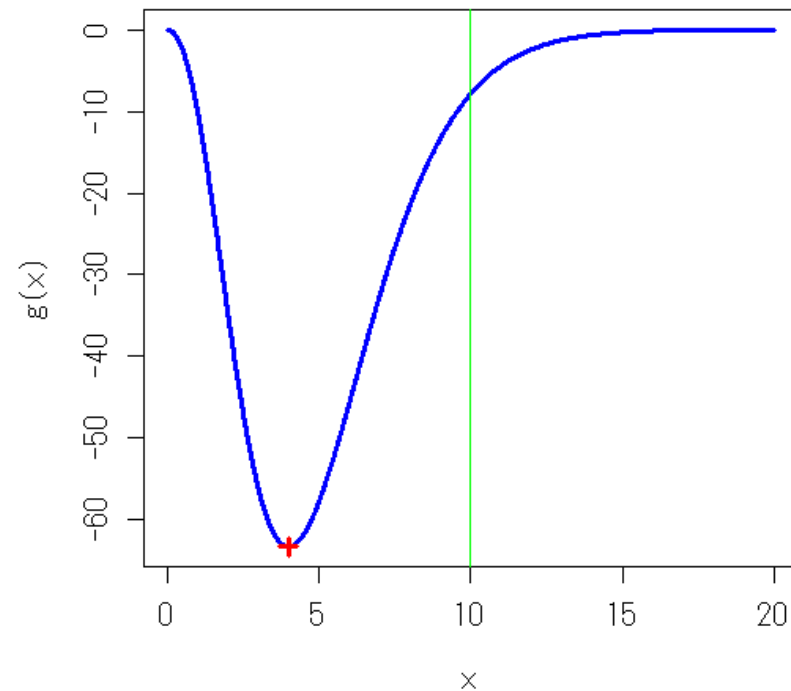
x0 <- 10
xmin <- optim(x0, g, method="BFGS",
              hessian=T)
xmin
```

```
$par
[1] 4.000001
$value
[1] -63.29885
$hessian
      [,1]
[1,] 13.18725
```

solution

Minimum value

Hessian (second derivative)



Once you describe a likelihood function, the computer calculates the MLE and its variance.

# Logistic regression of age at sexual maturity

simulated data `mature_age.txt`

```
read.table("mature_age.txt")->mature_age  
round(mature_age,3)
```

	age	maturity	
1	39.995	1	matured
2	12.427	0	immatured
3	25.069	1	matured
4	20.008	1	matured
5	19.420	1	matured
	. . . . .		

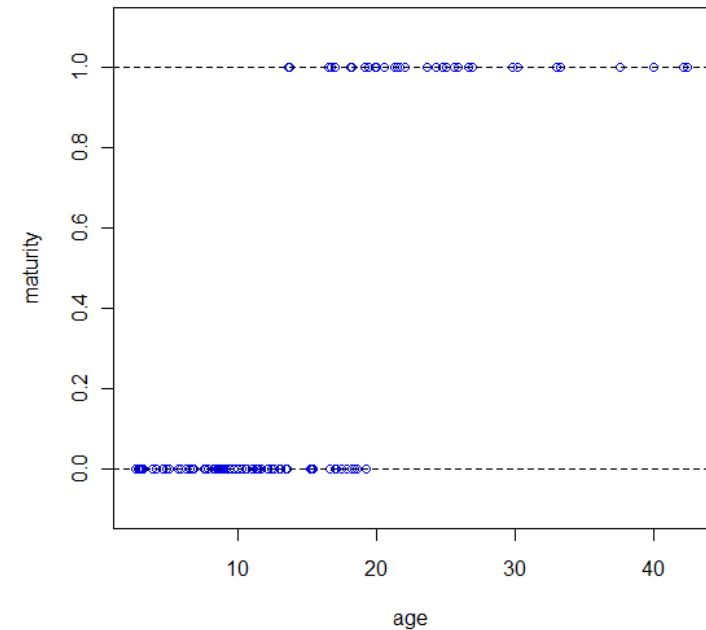
```
plot(maturity~age,mature_age)
```

Generalized linear model with binomial distribution

```
maturity.glm <-  
  glm(maturity~age,binomial,mature_age)
```

```
summary(maturity.glm)
```

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-10.2050	2.4413	-4.18	2.91e-05	***
age	0.5730	0.1377	4.16	3.18e-05	***



$$p = \frac{1}{1 + \exp(10.205 - 0.573x)}$$

# Logit transformation and reparametrization

Modeling the logit-transformed variable

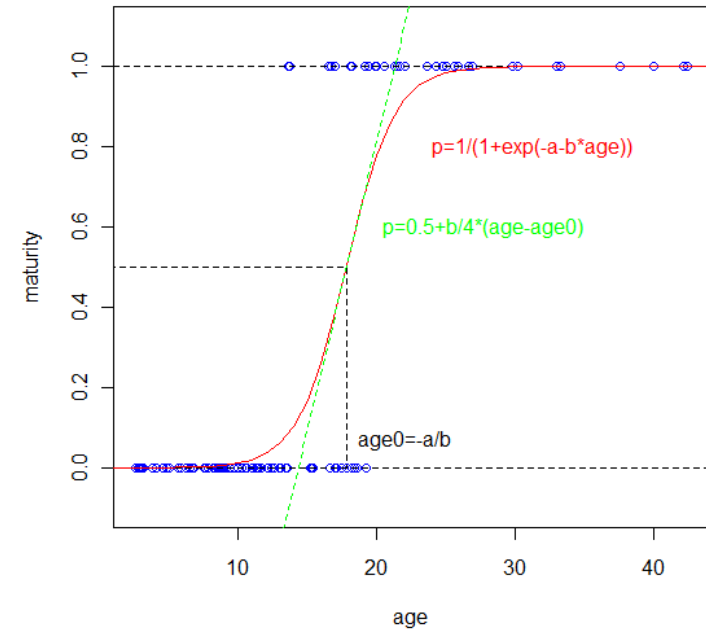
$$\log \frac{p}{1-p} = a + bx$$

$$p = \frac{1}{1 + \exp(-a - bx)}$$

Logistic function

Characterization

- $p = 0.5$  , when  $x = -\frac{a}{b}$
- Well approximated by a line  $p = 0.5 + \frac{b}{4} \left( x + \frac{a}{b} \right)$  at  $\left( -\frac{a}{b}, 0.5 \right)$



Reparametrization  $\text{age0} = -a/b$ ,  $c = b/4$  matches biological characters.

$$p = \frac{1}{1 + \exp(-4c(x - \text{age0}))}$$

# Maximum likelihood inference by R

```
nlike.glm <- function(theta,data){  
  c <- theta[1]; age0 <- theta[2]  
  n <- dim(data)[1]  
  x <- data[,1]; y <- data[,2]  
  p <- 1/(1+exp(-4*c*(x-age0)))  
  loglike <- sum(dbinom(y,size=1,prob=p,log=T))  
  return(-loglike)  
}  
  
theta0 <- c(0,0)  
theta_est <- optim(theta0,nlike.glm,data=mature_age,hessian=T)  
theta_est  
est <- theta_est$par  
se <- sqrt(diag(solve(theta_est$hessian)))  
mle <-  
data.frame(estimate=est,standard_error=se,row.names=c("c","age0"))  
round(mle,4)
```

	estimate	standard_error
c	0.1433	0.0344
age0	17.8097	0.6923



# Fitting K80 model to mtDNA sequences of human and chimpanzee

```
mtCDNA_human.nuc
```

```
CTACCCGCCGCGAGTACTGATCATTCTATTTCCCCCTCTATTGATCCCCACCTCCAAATAT
CTCATCAACAACCGACTAATTACCACCCAACAATGACTAATCAAACCTCAAAACAA
ATGATAGCCATACACAACACTAAAGGACGAACCTGATCTCTTATACTAGTATCCTTAATC
. . . . .
```

```
mtCDNA_chimp.nuc
```

```
TTACCCGCCGCGAGTACTAATCATTCTATTCCCCCTCTACTGGTCCCCACTTCTAAACAT
CTCATCAACAACCGACTAATTACCACCCAACAATGACTAATTCAAACCTCAAAACAA
ATAATAACTATACACAGCACTAAAGGACGAACCTGATCTCTCATACTAGTATCCTTAATC
. . . . .
```

```
scan("mtCDNA_human.nuc", what=" ") -> human
human <- strsplit(human, split=" ")
human <- unlist(human)
```

```
[1] "C" "T" "A" "C" "C" "C" "G" "C" "C" "G" "C" "A" "G" "T" "A" "C"
[17] "T" "G" "A" "T" "C" "A" "T" "T" "C" "T" "A" "T" "T" "T" "C" "C"
[33] "C" "C" "C" "T" "C" "T" "A" "T" "T" "G" "A" "T" "C" "C" "C" "C"
[49] "A" "C" "C" "T" "C" "C" "A" "A" "A" "T" "A" "T" "C" "T" "C" "A"
```

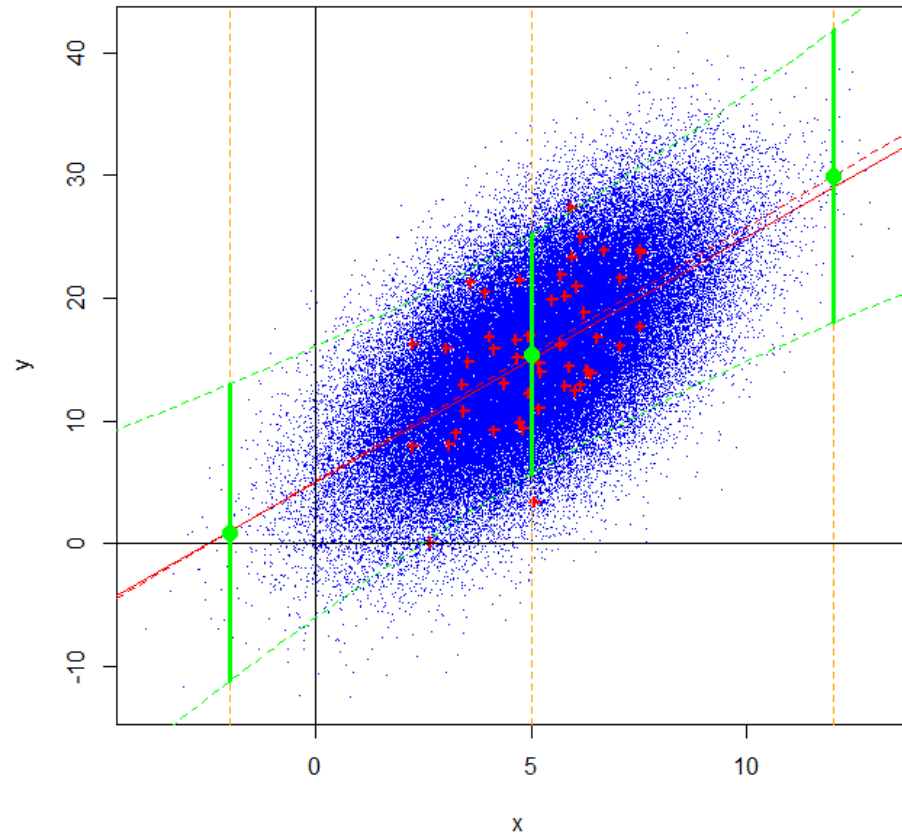
```
table(human, chimp)
```

	chimp			
human	A	C	G	T
A	2954	17	141	16
C	18	3163	4	374
G	165	5	1110	2
T	15	310	2	2411

	estimate	standard error
t	0.0037	0.0002
alpha	27.7425	1.9271

Kimura80\_pair\_ML.R

# Precision of the prediction and the complexity of the model



Complex models may improve the fitting to the data. However, the variance of the estimates become large, if the model include too many parameters beyond the amount of information in the data.

# Model selection based on the precision of the prediction

Variable selection of multiple regression

$$y_i = a + b_1 x_{1i} + \dots + b_k x_{ki} + \varepsilon_i \quad (i = 1, \dots, n)$$

Final prediction error **FPE** :

$$\text{FPE} = \frac{n + k + 1}{n - k - 1} \hat{\sigma}^2$$
$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n \left( y_i - \hat{a} - \hat{b}_1 x_{1i} - \dots - \hat{b}_k x_{ki} \right)^2$$

Prediction error is obtained by discounting the sum of squared residuals

By considering the predictive power for general models, Akaike (1974) derived a criteria defined by

$$\text{AIC} = -2 \times (\text{maximum likelihood value}) + 2 \times (\text{number of parameters})$$

In the special case of regression analysis,

$$\text{AIC} = n \log \hat{\sigma}^2 + 2(k + 2) \approx \log \text{FPE} - n \log n$$

A large, solid dark blue circle is centered on the page. Inside the circle, the text "Plot trees by ape" is written in white, bold, sans-serif font, centered both horizontally and vertically.

**Plot trees by  
ape**

# Read newick file

```
library(ape)
read.tree("desaturase_tree.nwk")->tree
summary(tree)
```

Phylogenetic tree: tree

Number of tips: 22

Number of nodes: 20

Branch lengths:

mean: 0.122944

variance: 0.02444867

distribution summary:

Min.	1st Qu.	Median	3rd Qu.	Max.
0.00000000	0.02457463	0.05800963	0.16716395	0.66505495

No root edge.

First ten tip labels: HzeZ9\_18

TniZ9\_18

BmoZ9\_18

EpoZ9\_18

OfuZ9\_18

OnuZ9\_18

EpoZ9\_16

PocZ9\_16

AveZ9\_16

HzeZ9\_16

No node labels.

# Look at the content

```
names(tree)
```

```
[1] "edge"          "edge.length" "Nnode"        "tip.label"
```

```
tree$edge
```

```
      [,1] [,2]  
[1,]    23    24  
[2,]    24    25  
[3,]    25    26  
  . . .
```

```
tree$edge.length
```

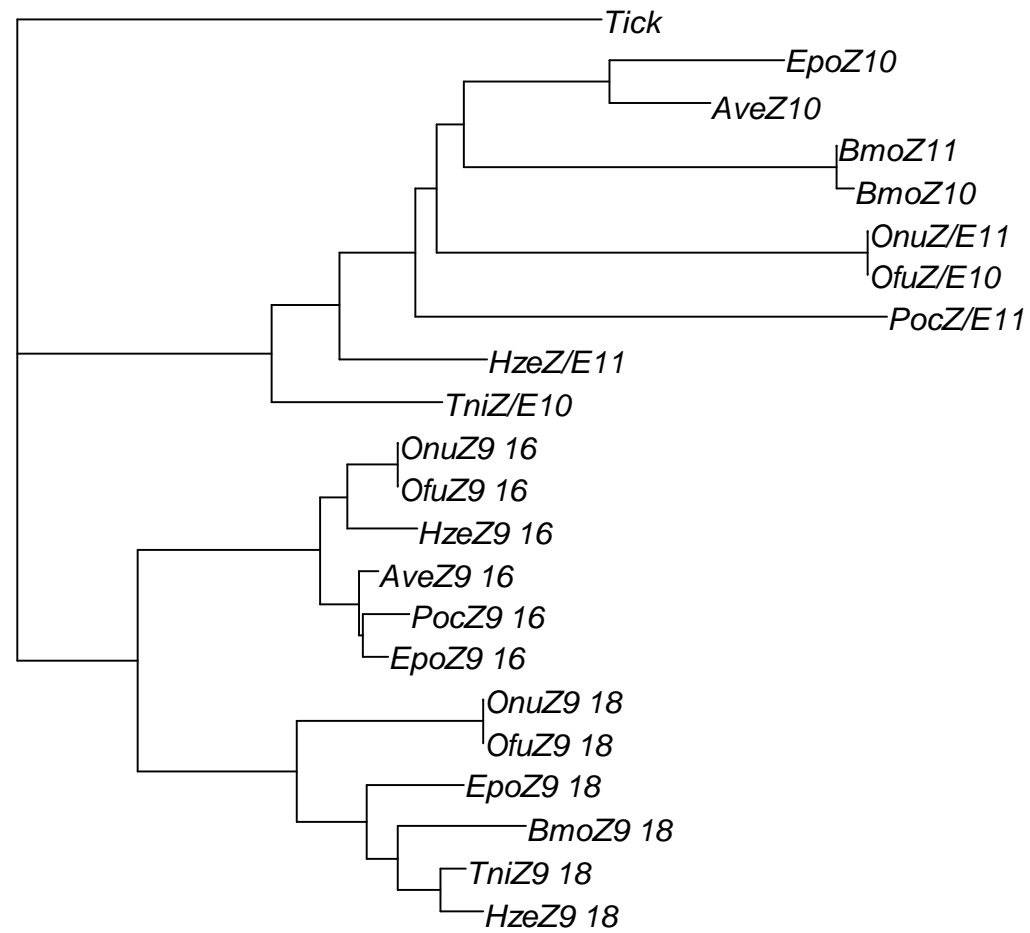
```
[1] 0.13716450 0.18089948 0.08089457 0.03392684 0.04898512 0.04775221  
[7] 0.02881577 0.14738616 0.11048854 0.21265592 0.00000000 0.00000000  
  . . .
```

```
tree$tip.label
```

```
[1] "HzeZ9_18" "TniZ9_18" "BmoZ9_18" "EpoZ9_18" "OfuZ9_18" "OnuZ9_18"  
[7] "EpoZ9_16" "PocZ9_16" "AveZ9_16" "HzeZ9_16" "OfuZ9_16" "OnuZ9_16"  
[13] "TniZ/E10" "HzeZ/E11" "PocZ/E11" "OfuZ/E10" "OnuZ/E11" "BmoZ10"  
[19] "BmoZ11"   "AveZ10"   "EpoZ10"   "Tick"
```

## Plot the tree

```
plot(tree)
```



# Check the structure of tip-labels and node labels

```
tree0 <- tree  
summary(tree0)
```

```
Phylogenetic tree: tree
```

```
Number of tips: 22  
Number of nodes: 20  
Branch lengths:  
  mean: 0.122944  
  variance: 0.02444867  
  distribution summary:  
      Min.      1st Qu.      Median      3rd Qu.      Max.  
0.00000000 0.02457463 0.05800963 0.16716395 0.66505495  
. . .
```

```
tree0$tip.label <- 1:22  
tree0$node.label <- 22+1:20
```

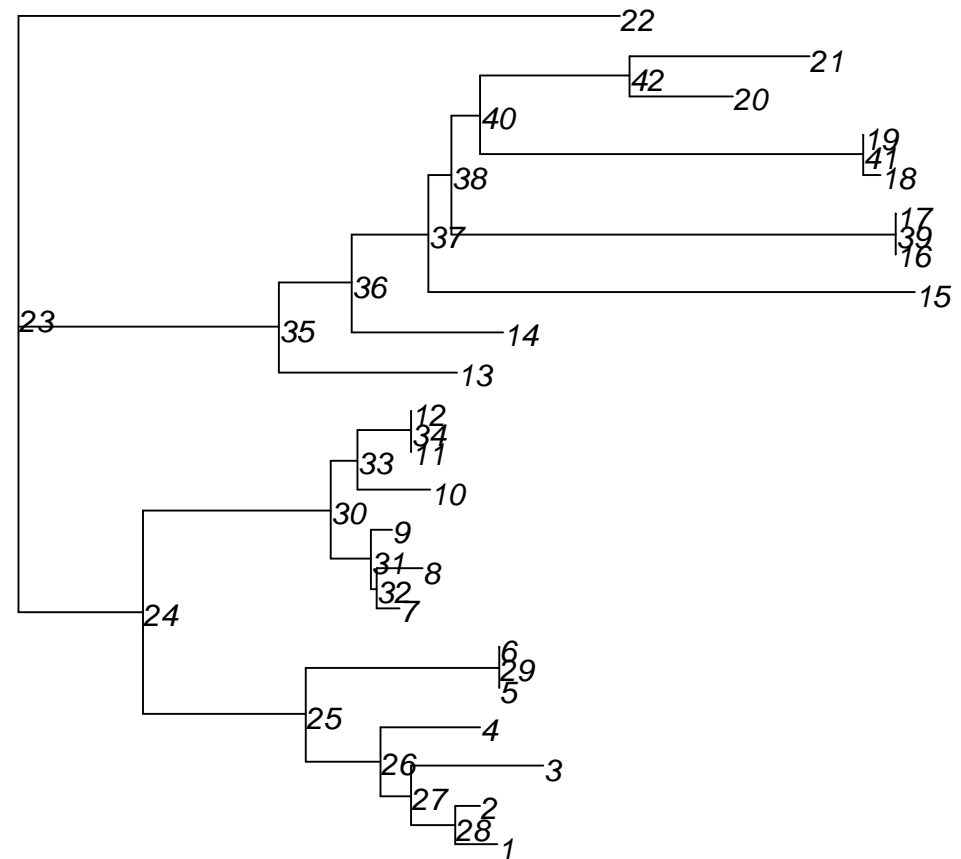


# Check the structure of edges

```
plot(tree0, show.node.label=T)
```

```
tree$edge
```

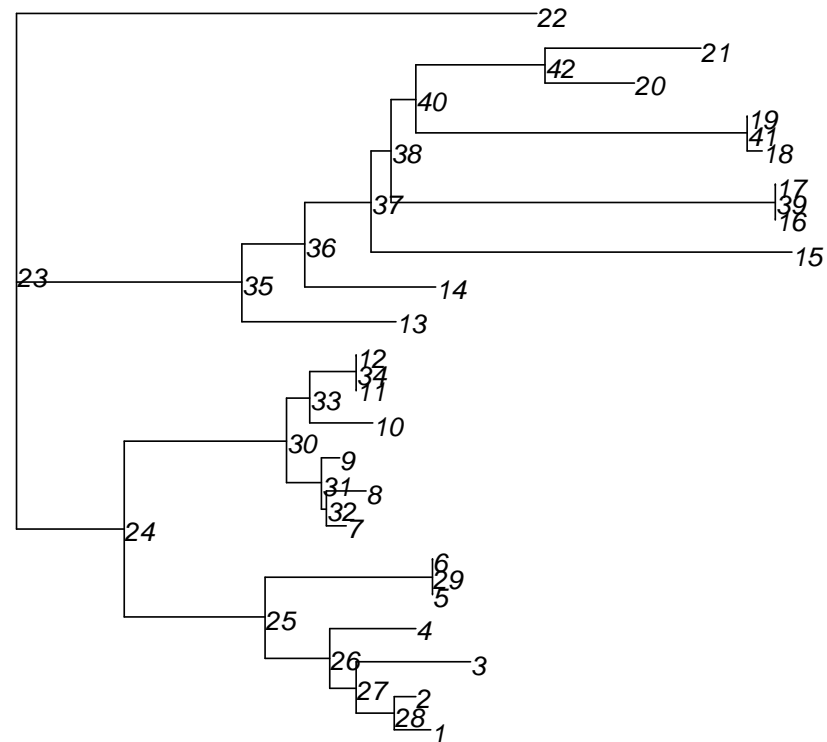
	[,1]	[,2]
[1,]	23	24
[2,]	24	25
[3,]	25	26
[4,]	26	27
[5,]	27	28
[6,]	28	1
[7,]	28	2
[8,]	27	3
[9,]	26	4
[10,]	25	29
[11,]	29	5
[12,]	29	6
[13,]	24	30
[14,]	30	31
[15,]	31	32
[16,]	32	7
[17,]	32	8
[18,]	31	9
. . . . .		



## Define the colors of edges for the three groups

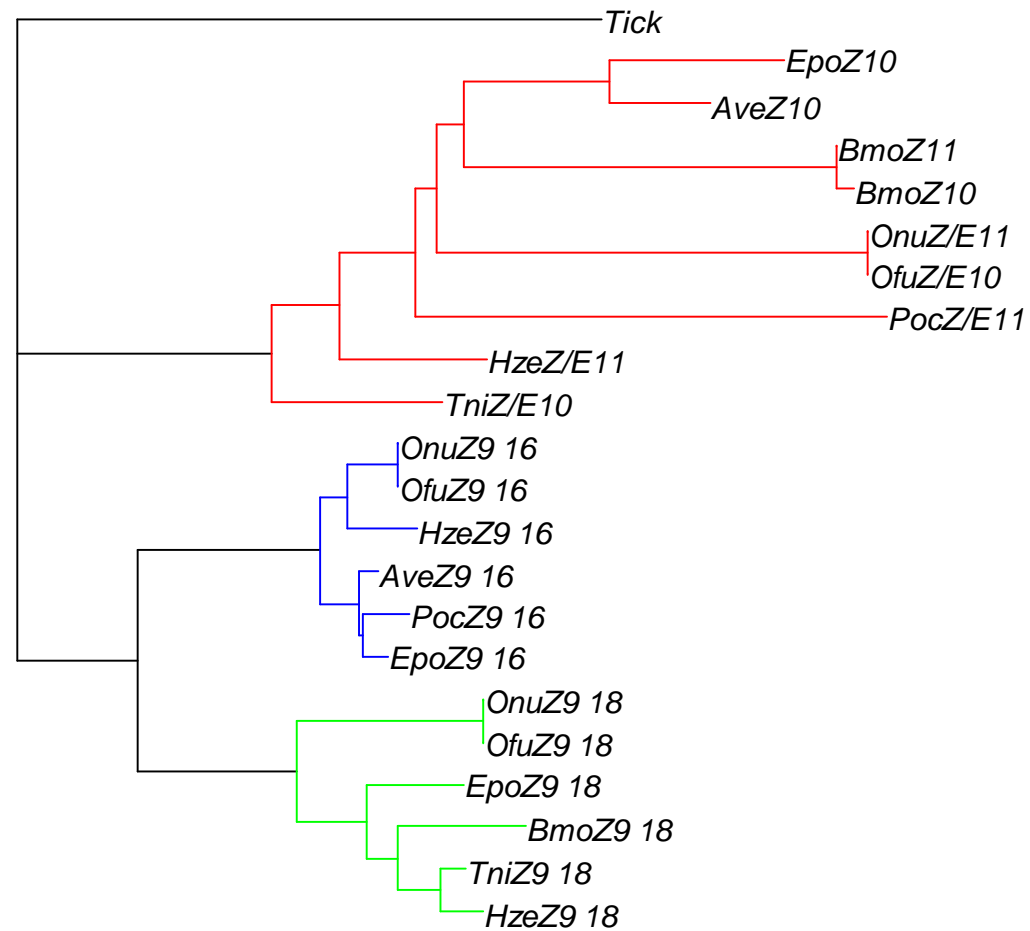
```
edges1 <- which.edge(tree, group=tree$tip.label[1:6])  
edges2 <- which.edge(tree, group=tree$tip.label[7:12])  
edges3 <- which.edge(tree, group=tree$tip.label[13:21])
```

```
edge.colors <- rep("black",dim(tree$edge)[1])  
edge.colors[edges1] <- "green"  
edge.colors[edges2] <- "blue"  
edge.colors[edges3] <- "red"
```



# Plot the tree with colors for the three groups

```
plot(tree,edge.color=edge.colors)
```



# Assignment 1

Construct the phylogenetic tree of desaturase by the models; LG+G, LG, WAG+G, WAG. Compare the values of AICc and BIC, and interpret the difference of the phylogenetic trees. Please also try ape.

Please send the word file / pdf file named **agri1\_name.doc /  
agri1\_name.pdf** to:

Hirohisa Kishino (kishino@lbm.ab.a.u-tokyo.ac.jp ).

Here, “name” should be replaced by your name.

Deadline: 26 May 2019 (Sunday)